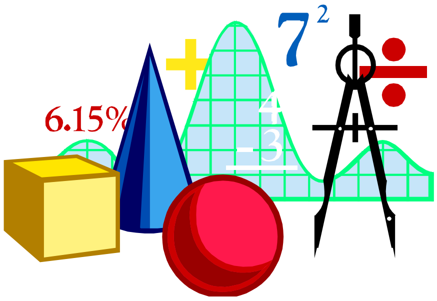
ALGEBRA

SECOND TERM

PREPARATORY ONE

PREPARED BY

Mr. MAHMOUD



Lesson (1):
Repeated multiplication

☞ $١^{٢} = ١ \times 1 = 1$

☞ $٢^{٢} = ٢ \times 2 = 4$

☞ $٣^{٢} = ٣ \times 3 = 9$

☞ $٤^{٢} = ٤ \times 4 = 16$

☞ $٥^{٢} = ٥ \times 5 = 25$

☞ $٦^{٢} = ٦ \times 6 = 36$

☞ $٧^{٢} = ٧ \times 7 = 49$

☞ $٨^{٢} = ٨ \times 8 = 64$

☞ $٩^{٢} = ٩ \times 9 = 81$

☞ $١٠^{٢} = ١٠ \times 10 = 100$

These numbers are called square numbers.

☞ $١^{٣} = ١ \times 1 \times 1 = 1$

☞ $٢^{٣} = ٢ \times 2 \times 2 = 8$

☞ $٣^{٣} = ٣ \times 3 \times 3 = 27$

☞ $٤^{٣} = ٤ \times 4 \times 4 = 64$

☞ ٥ ٣

☞ ٦ ٣

☞ $٧^{٣} = ٧ \times 7 \times 7 = 343$

☞ $٨^{٣} = ٨ \times 8 \times 8 = 512$

☞ $٩^{٣} = ٩ \times 9 \times 9 = 729$

☞ ١٠

These numbers are called cube numbers.

$$\text{Side length} \underset{\sqrt{A}}{\overset{s^2}{\rightleftarrows}} \text{Area}$$

Ex(1): A square whose side length 5 cm. Find its area.

....

$$\text{Edge length} \underset{\sqrt[3]{V}}{\overset{s^3}{\Leftrightarrow}} \text{Volume}$$

Ex(2): A cube whose edge length 5 cm. Find its volume.

....

Remarks:

☞ If $\frac{a}{b}$ is a rational number, then $\left(\frac{a}{b}\right)^0 = 1$ where $a \neq 0$

"Any number of power zero = 1 except zero"

☞ $\left(\frac{-a}{b}\right)^m = \left(\frac{a}{b}\right)^m$ when m is an even number.

☞ $\left(\frac{-a}{b}\right)^m = -\left(\frac{a}{b}\right)^m$ when m is an odd number.

[1] Choose the correct answer

١.	The multiplicative inverse of the number $\left(\frac{2}{5}\right)^0 =$			
	(a) $\frac{5}{2}$	(b) $-\frac{2}{5}$	(c) 1	(d) 0
٢.	The additive inverse of the number $(-3)^0$ is			
	(a) 1	(b) – 3	(c) 3	(d) $-(3)^0$
٣.	The multiplicative inverse of the number $(-1)^3$ is			
	(a) $(-1)^3$	(b) $(-1)^2$	(c) 1^3	(d) 1^2
٤.	The additive inverse of the number $\left(-\frac{2}{5}\right)^2$ is			
	(a) $\frac{4}{25}$	(b) $-\frac{4}{25}$	(c) $\frac{25}{4}$	(d) $-\frac{25}{4}$
٥.	$\left(\frac{1}{4}\right)^0 + \frac{1}{4} =$			
	(a) $\frac{1}{4}$	(b) $\frac{3}{4}$	(c) $\frac{5}{4}$	(d) $\frac{2}{4}$
٦.	$\left(\frac{5}{3}\right)^2 \times \left(\frac{3}{5}\right)^0 =$			
	(a) $\frac{5}{3}$	(b) $\frac{25}{9}$	(c) 0	(d) 1
٧.	If $x = y$, then $\left(\frac{3}{5}\right)^{x-y} =$			
	(a) $\frac{3}{5}$	(b) $\frac{5}{3}$	(c) 1	(d) 0
٨.	$\left(\frac{a}{b}\right)^2 \times \frac{b^2}{a^2} =$ .... (where $ab \neq 0$)			
	(a) ab	(b) $\left(\frac{a}{b}\right)^4$	(c) $(ab)^0$	(d) $\frac{a}{b}$

٩.	If $x = -\frac{1}{2}$ and y = 3 , then x^y = ……… (a) $\frac{1}{8}$ (b) $-\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$
١٠.	If $y^{26} + y^{27} = 0$, then y = ……… (a) 1 (b) – 1 (c) 2 (d) – 2

[2] Complete:

(1) $\frac{8}{27} = \left(\frac{2}{3}\right)^{\cdots\cdots}$

(2) $\frac{9}{16} = \left(\frac{3}{4}\right)^{\cdots\cdots}$

(3) $0.027 = \left(\frac{3}{10}\right)^{\cdots\cdots}$

(4) $64\% = \left(\frac{4}{5}\right)^{\cdots\cdots}$

(5) If $\frac{x}{y} = \frac{-2}{5}$, then $\left(\frac{x}{y}\right)^3$ = ……………………

(6) $2^2 + 2^2 = 2^{\cdots}$

(7) $\left(\frac{-1}{2}\right)^3 - \left(\frac{-1}{2}\right)^2$ = ………………………………………

(8) $\frac{3}{4}$, $\frac{9}{16}$, $\frac{27}{64}$, ………, ……… (in the same sequence)

[3] Calculate each of the following in the simplest form:

(1) $\left(\frac{1}{2}\right)^3$ = …………………………………………

(2) $\left(\frac{1}{3}\right)^4$ = …………………………………………

(3) $\left(\frac{-1}{7}\right)^3$ = …………………………………………

(4) $\left(\frac{-3}{4}\right)^4 =$ ……………………………………………

(5) $\left(\frac{5}{9}\right)^0 =$ ……………………………………………

(6) $\left(1\frac{1}{5}\right)^2 =$ ……………………………………………

(7) $(0.04)^2 =$ ……………………………………………

(8) $(-3.2)^2 =$ ……………………………………………

(9) $\left(1 - 1\frac{2}{3}\right)^2 =$ ……………………………………………

١٠

Lesson (2):
Non-negative integer power

Rules:

☞ $\left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$

☞ $\left(\frac{a}{b}\right)^n \div \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n-m}$

☞ $\left[\left(\frac{a}{b}\right)^n\right]^m = \left(\frac{a}{b}\right)^{n\times m}$

[1] Calculate each of the following, then put the resutl in the simplest form:

(1) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2 =$ ……………………………………

(2) $\left(\frac{-2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2 =$ ……………………………………

(3) $\left(\frac{1}{5}\right) \times \left(\frac{-1}{5}\right)^4 =$ ……………………………………

(4) $\left(\frac{1}{6}\right)^9 \div \left(\frac{1}{6}\right)^8 =$ ……………………………………

(5) $\left(\frac{2}{7}\right)^5 \div \left(\frac{2}{7}\right)^3 =$ ……………………………………

(6) $\left(\frac{-3}{5}\right)^7 \div \left(\frac{3}{5}\right)^5 =$ ……………………………………

(7) $\left(-\frac{5}{2}\right)^2 \div 2\frac{1}{2} =$ ……………………………………

(8) $\left(\frac{4}{5}\right)^8 \div \left(\frac{4}{5}\right)^6 \times \left(\frac{4}{5}\right) =$ ....

[2] Calculate each of the following, then put the result in the simplest form:

(1) $\frac{3^7 \times 3^3}{3^6} =$ ....

(2) $\frac{2^6 \times 2}{2^3 \times 2^4} =$ ....

(3) $\frac{(-5)^4 \times 5^2}{5^3} =$ ....

(4) $\frac{(-2)^5 \times 2^4}{(-2)^3 \times 2^2} =$ ....

(5) $\frac{(-3)^5 \times (-2)^7}{(-3)^3 \times (-2)^5} =$ ....

(6) $\frac{x^2 \times x^3 \times x^4}{x^7 \times x} =$ ....

(7) $\frac{x^4 \times y^3 \times x^5}{x^6 \times y^2} =$ ....

(8) $\left(\frac{a\,b}{c}\right)^5 =$ ....

(9) $\left(\frac{5x}{3y}\right)^2 =$ ....

(10) $\left(\frac{-2a\,b}{3c}\right)^4 =$ ....

(11) $\left(\frac{x^2}{y^3}\right)^2 =$ ……………………………………………

(12) $\left(\frac{a^3b^2}{c^5}\right)^3 =$ ……………………………………………

(13) $\left(\frac{-c^2}{d}\right)^3 =$ ……………………………………………

(14) $\left(\frac{-x^3}{y^2}\right)^2 =$ ……………………………………………

(15) $\left[\left(\frac{1}{2}\right)^2\right]^2 =$ ……………………………………………

(16) $\left[\left(\frac{-3}{2}\right)^2\right]^5 =$ ……………………………………………

(17) $\left[\left(2\frac{1}{2}\right)^3\right]^2 =$ ……………………………………………

(18) $\left(\frac{3}{5}\right)^{10} \times \left(\frac{5}{3}\right)^{10} =$ ……………………………………………

(19) $\left[\left(\frac{2}{7}\right)^2\right]^3 \times \left(\frac{7}{2}\right)^6 =$ ……………………………………………

(20) $\left(2\frac{1}{2}\right)^2 \times \left(\frac{-2}{5}\right)^2 =$ ……………………………………………

Lesson (3):
Negative integer power

☞ We studied the multiplicative inverse of rational numbers, the multiplicative inverse of $\frac{1}{2}$ is $\frac{2}{1} = 2$.

☞ We can write the fractions in the decimal form such as:
$\frac{1}{2} = 0.5$ and $\frac{1}{4} = 0.25$

☞ And we can write the fraction in the whole form by changing the sign of its power such as:

$0.5 = \frac{1}{2^1} = 2^{-1}$ $\qquad$ $0.25 = \frac{1}{4^1} = 4^{-1}$

$0.\dot{3} = \frac{1}{3} = 3^{-1}$ $\qquad$ $\frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$0.1 = \frac{1}{10} = 10^{-1}$ $\qquad$ $0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$

$0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

☞ $2^1 \xrightarrow{reciprocal} 2^{-1}$, $\quad 2 \times 2^{-1} = 1$

$4^1 \xrightarrow{reciprocal} 4^{-1}$, $\quad 4 \times 4^{-1} = 1$

$2^3 \xrightarrow{reciprocal} 2^{-3}$, $\quad 2^3 \times 2^{-3} = 1$

☞ If we want to get the multiplicative inverse of any rational number, change the sign of its power.

[1] Complete:

(1) If $a = \frac{2}{3}$, then $a^{-1} =$ ....

(2) If $a = 7^x$, $b = 7^{-x}$, then $a \times b =$ ......

[2] Evaluate each of the following:

(1) $4^{-1} =$ ……………………………………

(2) $5^{-2} =$ ……………………………………

(3) $\left(\frac{1}{2}\right)^{-1} =$ ……………………………………

(4) $\left(-\frac{2}{3}\right)^{-2} =$ ……………………………………

(5) $(0.2)^{-2} =$ ……………………………………

(6) $(1.2)^{-1} =$ ……………………………………

[3] Calculate each of the following, then put the result in the simplest form:

(1) $3^{7} \times 3^{-3} =$ ……………………………………

(2) $2^{-2} \times 2^{-3} =$ ……………………………………

(3) $\frac{3}{3^{-2}} =$ ……………………………………

(4) $\frac{6^{-2}}{6^{-3}} =$ ……………………………………

[4] Calculate each of the following, then put the result in the simplest form:

(1) $\frac{8 \times 8^{-2}}{8^{-3}} =$ ……………………………………

(2) $\frac{7^{-2} \times 7^{5}}{7^{3}} =$ ……………………………………

(3) $\dfrac{2^5 \times 2^{-2}}{2^{-4} \times 2^3} =$ ………………………………………

(4) $(5^{-1})^{-3} =$ ………………………………………

(5) $(3^{-2})^2 =$ ………………………………………

(6) $(0.25)^{-2} =$ ………………………………………

(7) $(2^{-1} \times 2^{-2})^3 =$ ………………………………………

(8) $\left(\dfrac{3^{-1}}{3}\right)^2 =$ ………………………………………

(9) $\left(\dfrac{8^4}{8^{-4}}\right)^0 =$ ………………………………………

(10) $\dfrac{(3^{-2})^3}{3^{-2} \times 3^{-6}} =$ ………………………………………

(11) $\left(\dfrac{9^3 \times 9}{9^5}\right)^{-3} =$ ………………………………………

[5] Simplify each of the following:

(1) $7x^{-1} =$ ………………………………………

(2) $x^{-1}y^2 =$ ………………………………………

(3) $a^{-2}b^{-3} =$ ………………………………………

(4) $x^3 \times x^{-5} =$ ………………………………………

(5) $x^3 \times x^{-2} \times x^{-1} =$ ………………………………………

(6) $\frac{c^{-5}}{c^{2}} =$ ....

(7) $x^{7} \div x^{-5} =$ ....

(8) $\left(a^{-2}\right)^{3} =$ ....

(9) $\left(x^{2}\right)^{-3} \times \left(x^{-3}\right)^{-2} =$ ....

(10) $\left(b^{-1}\right)^{-3} = b^{\cdots}$

(11) $2x^{-3} = \frac{2}{\cdots\cdots}$

(12) $\left(3x^{-1}\right)^{2} = 9x^{\cdots} = \frac{9}{\cdots\cdots}$

(13) $\left(3a^{2}\right)^{-1} = \frac{1}{\cdots\cdots}$

(14) $2x^{-2}y^{-3} = \frac{2}{\cdots\cdots}$

(15) $2^{10} \times 2^{-10} = 3^{\cdots}$

[6] Choose the correct answer:

(1) $3^{2} \times 3^{5} =$ ....
(a) 3^{7} (b) 3^{3} (c) 3^{10} (d) 3^{25}

(2) $5^{2} + 5^{2} =$ ....
(a) 10^{2} (b) 10^{4} (c) 5^{4} (d) 50

(3) $3^{5} \times 2^{5} =$ ....
(a) 5^{10} (b) 6^{10} (c) 6^{5} (d) 6^{25}

(4) $(5a)^{0} =$ ...., $a \neq 0$
(a) 5 (b) a (c) $5a$ (d) 1

(5) $3^{(2^3)} = \dots\dots$

(a) 3^6 (b) 3^5 (c) 3^8 (d) 3^{23}

(6) $(5^2)^3 = \dots\dots$

(a) 5^6 (b) 5^5 (c) 5^{23} (d) 5

(7) $3^{10} + 3^{10} + 3^{10} = \dots\dots$

(a) 3^{10} (b) 3^{30} (c) 3^{11} (d) 9^{10}

(8) $4^x + 4^x + 4^x + 4^x = \dots\dots$

(a) 4^{x+4} (b) 4^{4x} (c) 4^{x+1} (d) $4x^4$

(9) $\frac{(3^2)^5}{(3^5)^2} = \dots\dots$

(a) 3^{10} (b) 3^{52} (c) 3^{25} (d) 1

(10) $(2y)^3 = \dots\dots$

(a) $2y^3$ (b) $8y$ (c) $8y^3$ (d) $23y$

(11) $(b^3)^4 = \dots\dots$

(a) b^{34} (b) $b^3 \times b^3 \times b^3$ (c) b^7 (d) $b^4 \times b^4 \times b^4$

(12) The quarter of the number 4^{20} is ………

(a) 4^5 (b) 4^{10} (c) 4^{19} (d) 2^{10}

(13) If $a^{-1} = \frac{2}{3}$, then $a = \dots\dots$

(a) $\frac{-2}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) 1

(14) If $a = 7^x$ and $b = 7^{-x}$, then $a \times b = \dots\dots$

(a) 7^{2x} (b) 49^{2x} (c) 1 (d) 0

(15) $\frac{6a^2x^4}{2a^3x^3} = \dots\dots$

(a) $3ax$ (b) $3a^5x^7$ (c) $\frac{3x}{a}$ (d) $\frac{3}{ax}$

(16) $\frac{(-2s^2t)^3}{(-4st^2)^2} = \dots\dots$

(a) $\frac{-s^3}{2t}$ (b) $\frac{-s^4}{2t}$ (c) $\frac{s^5}{2t^2}$ (d) $\frac{s^4}{t}$

(17) If $a^x = 2$ and $a^{-y} = 3$, then $a^{x-y} =$ ....

(a) 1 (b) -1 (c) $\frac{2}{3}$ (d) 6

(18) If $x\,y^{-1} = \frac{1}{2}$, then $\frac{y}{x} =$ ....

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) 1 (d) 2

(19) $3^{-1} + 3^{-1} + 3^{-1} =$ ....

(a) 3^{-3} (b) 3^3 (c) 9^{-3} (d) 1

(20) The multiplicative inverse of 5^{-1} is ....

(a) $\frac{1}{5}$ (b) 5 (c) -5 (d) $\frac{-1}{5}$

(21) $\left(\frac{3}{5}\right)^2 \times \left(\frac{5}{3}\right)^{-2} =$ ....

(a) $\left(\frac{3}{5}\right)^4$ (b) 1 (c) $\left(\frac{3}{5}\right)^{-4}$ (d) 0

[7] Calculate each of the following:

(1) $8 \times \left(\frac{1}{2}\right)^3 =$ ....

(2) $\left(\frac{-3}{4}\right)^2 \times \frac{8}{27} =$ ....

(3) $\left(\frac{-3}{5}\right)^3 \times \left(\frac{-25}{27}\right) =$ ....

(4) $\left(\frac{-5}{6}\right)^2 \div 3\frac{3}{4} =$ ....

(5) $2\frac{7}{9} \div \left(-1\frac{2}{3}\right)^2 =$ ....

[8] If $a = \frac{-1}{2}$, $b = 2$ and $c = \frac{3}{4}$. Find the numerical value of $a^3b^2 + b^2c - 8abc$

....

....

....

....

[9] If $x = \frac{-1}{2}$, $y = \frac{3}{4}$ and $z = \frac{-3}{2}$. Find the numerical value of:

(1) x^3y^2 (2) $\frac{x^3}{y^2z^2}$

....

....

....

....

....

....

....

....

....

....

....

....

Lesson (4)

Scientific notation of the rational number

☞ The number is written in the standard form as $a \times 10^n$ where $1 \le |a| < 10$ and $n \in Z$.

[1] Which of the following numbers are in the standard form?

(1) 5.3×10^7 (2) 0.2×10^{-4}

(3) 0.025×10^8 (4) 7×10^{-4}

(5) 10×10^{-10} (6) 4.25×10

(7) 33.9×10^6 (8) -5.783×10^2

(9) -0.0003×10^3 (10) 3.912×10^{-2}

[2] Write each of the following numbers are in the standard form:

(1) ٦٠٠ ٠٠٠ = ...

(2) -٢٠ ٠٠٠ = ...

(3) ٧ million = ...

(4) ١٩ million = ...

(5) ٠,٠٠٠٦ = ...

(6) ٠,٠٠٠٠٥٣ = ...

(7) ٠,٠٠٠٨٦٤ = ...

(8) ٠,٤٢١ = ...

(9) ٥١٠ ٠٠٠ ٠٠٠ km٢ = ...

(10) ٦٨ × 10^5 = ...

(11) ٦٨ × 10^{-5} = ……………………………………

(12) ٧٢٠ × 10^{6} = ……………………………………

(13) ٧٥٠ × 10^{-9} = ……………………………………

(14) -٣٢,٤ × 10^{4} = ……………………………………

(15) ٠,٠٠٠٠٥ × 10^{15} = ……………………………………

(16) ٠,٠٠٠٢٠٢٠٥ × 10^{12} = ……………………………………

[3] Write the result of each of the following in the standard form:

(1) $(6.4 \times 10^{8}) \times (1.5 \times 10^{5})$ = ……………………………………………………………………………

(2) $(8.2 \times 10^{7}) \times (2.1 \times 10^{-4})$ = ……………………………………………………………………………

(3) $(5.02 \times 10^{-4}) \times (0.1 \times 10^{-3})$ = ……………………………………………………………………

(4) $(3.8 \times 10^{8}) \div (1.9 \times 10^{6})$ = ……………………………………………………………………………

(5) $(125.5 \times 10^{-3}) \div (5 \times 10^{4})$ = ……………………………………………………………………

(6) $(3.8 \times 10^{5}) + (4.6 \times 10^{4})$ = ……………………………………………………………………………

(7) $(4.54 \times 10^{4}) + (3.76 \times 10^{3})$ = ……………………………………………………………………

(8) $(5.3 \times 10^{8}) - (0.8 \times 10^{7})$ = ……………………………………………………………………………

(9) $(2.65 \times 10^{-2}) + (6.34 \times 10^{-3})$ = ……………………………………………………………

[4] Choose the correct answer:

(1) ٣,٠٤ × 10^{7} = …………………

(a) 340 000 (b) 304 000 (c) 3 400 000 (d) 30 400 000

(2) ٢,٣٧ × 10^{-4} = …………………
(a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

(3) If 0.00079 = 7.9 × a, then a = ………
(a) 10^{3} (b) 10^{-3} (c) 10^{-4} (d) 10^{4}

(4) If 0.00000503 = m × 10^{-5}, then m = ………
(
a
)

[4] Find the value of n in each of the following:

(1) ٨٠٠ ٠٠٠ = ٨ × 10^{n} n = ………
٥
(2) ٠,٠٠٠٠٠٠٠٠٦ = ٦ × 10^{n} n = ………
٠
(3) ٣,٠٠٠٠٥٢ = ٥,٢ × 10^{n} n = ………

(4) ٠,٠٠٠٠٣٥٧ = ٣,٥٧ × 10^{n} n = ………

(5) ٦٢٩٣ = n × 10^{4} n = ………
(
b
)

٥
.
٠
٣

(
c
)

٥

Lesson (5)

Order of mathematical operations

(1) Perform the operations within parentheses.

(2) Evaluate the power.

(3) Perform × and ÷ from left to right.

(4) Perform + and - from left to right.

[1] Calculate the value of each of the following:

(1) ٣ + ١٢ ÷ 6
= ....
= ....
= ....

(2) ٢ × 6 - 4 ÷ 2
= ....
= ....
= ....

(3) ٤ × 7 - 3^2
= ....
= ....
= ....

(4) ١٤٤ – ٨ ÷ 2^3
= ....
= ....
= ....

(5) ١٩٦ ÷ $(7-5)^2$
= ....
= ....
= ....

(6) ٧ (٦^٢ ÷ 2 × 3)
= ....
= ....
= ....

(7) ١٢ (٢^٢) ÷ 24 + 3^2
= ....
= ....
= ....

(8) ٩ (٤^٢) ÷ 2^2 × 3
= ....
= ....
= ....

(9) 2 – [(7 – 3) – 2]
= ....
= ....
= ....

(10) [4 – (5 – 2)] – 1
= ....
= ....
= ....

(11) $3+[5+2(8\div 4)]$

= ....

= ....

= ....

(12) $2[(5^2+1)-(4^2-1)]$

= ....

= ....

= ....

(13) $5[(2^2-1)-(2^2-2)]$

= ....

= ....

= ....

(14) $\frac{15+7}{15-4}$

= ....

= ....

= ....

(15) $\frac{8+20-4}{8-4}$

= ....

= ....

= ....

(16) $\frac{5+2\times 5}{2^2+1}+5^2-5$

= ....

= ....

= ....

(17) $\frac{-4\times(-10)}{-9+7}$

= ....

= ....

= ....

(18) ٢ $\times 6 - 4 \div 2$

= ....

= ....

= ....

(1) If x = 3, what is the numerical value of the expression $2\left(\frac{5x+3}{4x-3}\right)$

....

....

(2) Evaluate: $16t \div (4s) + 3st$, for $t = 9$ and $s = 6$

....

....

(3) Simplify: $\frac{n}{2}(3n-6)+\frac{1}{3}(3+9n)$, then find its numerical value when $n = 1$.

....

....

....

Lesson (6)
The square root of a perfect square rational number

[1] Calculate the value of each of the following:

(1) $\sqrt{16}$ = ………… (2) $-\sqrt{25}$ = …………

(3) $\pm\sqrt{2500}$ = ………… (4) $\pm\sqrt{40000}$ = …………

(5) $\sqrt{\frac{9}{49}}$ = ………… (6) $\sqrt{6\frac{1}{4}}$ = …………

(7) $-\sqrt{4^2}$ = ………… (8) $\pm\sqrt{8^2}$ = …………

(9) $\sqrt{\left(\frac{81}{100}\right)^2}$ = ………… (10) $\sqrt{\left(-\frac{3}{4}\right)^2}$ = …………

(11) $\pm\sqrt{\frac{16\,b^8}{121\,h^2}}$ = ………… (12) $\sqrt{\frac{49\,a^4b^2}{9}}$ = …………

(13) $\sqrt{\frac{25\,x^2y^2}{36}}$ = ………… (14) $\sqrt{0.36}$ = …………

[2] Find each of the following in the simplest form:

(1) $\sqrt{9}+\sqrt{16}$ = ……………………………………………

(2) $\sqrt{36+64}$ = ……………………………………………

(3) $\sqrt{25-9}$ = ……………………………………………

(4) $\sqrt{3^2+4^2}$ = ……………………………………………

(5) $\sqrt{\frac{5^4\times 5^3}{5^5}}$ = ……………………………………………

(6) $\sqrt{\frac{9}{16}+1}$ = ……………………………………………

(7) $\sqrt{\left(\frac{1}{2}\right)^2\div\left(\frac{1}{5}\right)^2}$ = ……………………………………………

(8) $\sqrt{\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{3}\right)^4}$ = ...

(9) $\frac{3}{4} \times \sqrt{\frac{16}{9}}$ = ...

(10) $\sqrt{\frac{81}{49}} \times \frac{14}{27}$ = ...

(11) $\sqrt{\frac{9}{4}} - \frac{3}{2} + \left(\frac{3}{2}\right)^{zero}$ = ...

[3] Complete:

(1) The multiplicative inverse of the number $\sqrt{\frac{4}{25}}$ in the simplest form is ....

(2) The multiplicative inverse of the number $\sqrt{0.49}$ in the simplest form is ....

(3) The multiplicative inverse of the rational number $\sqrt{\frac{10}{2.5}}$ in the simplest form is ....

(4) The additive inverse of the number $-\sqrt{\frac{9}{16}}$ in the simplest form is ....

(5) The multiplicative inverse of the number $\sqrt{\frac{9}{16}}$ in the simplest form is ....

[4] Simplify:

(1) $\left(\frac{3}{4}\right)^{zero} \times \sqrt{\frac{81}{64}} \times \left(\frac{-2}{3}\right)^3$ = ...

(2) $\left(\frac{-2}{5}\right)^2 \times \left(\frac{-3}{5}\right)^0 \times \sqrt{6\frac{1}{4}}$ = ...

(3) $\sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^{zero} \times \left(\frac{-2}{7}\right)^{2} =$ ....

(4) $\frac{2}{5} \times \sqrt{\frac{9}{16}} \div \left(-\frac{1}{2}\right)^{3} =$ ....

(5) $\left(\frac{-1}{3}\right)^{2} + \sqrt{\frac{64}{81}} - \left(\frac{3}{4}\right)^{zero} =$ ....

[4] Choose the correct answer:

(1) $\sqrt{1\frac{9}{16}} =$ ....

(a) $1\frac{3}{4}$ (b) $-1\frac{3}{4}$ (c) $1\frac{1}{4}$ (d) $-1\frac{1}{4}$

(2) $\sqrt{10^2 - 6^2} =$ ....

(a) 4 (b) 8 (c) ± 4 (d) ± 8

(3) $\sqrt{18 \times 10 \times 10 \times 18} =$ ....

(a) 18 (b) 180 (c) 10 (d) 100

(4) $\sqrt{\sqrt{81}} =$ ....

(a) 81 (b) 27 (c) 9 (d) 3

(5) If $\frac{x}{2} = \frac{8}{x}$, then $x =$ ....

(a) 4 (b) - 4 (c) ± 4 (d) 16

(6) If $x = \sqrt{\frac{1}{4}}$, then $x^3 =$ ....

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{64}$

Lesson (7)
Solving equations
of the first degree in one unknown in Q

[1] Find the solution set of each of the following equations:

(1) $x - 7 = 3$, where $x \in N$

....
....
....
....

(2) $x + 17 = 13$, where $x \in N$

....
....
....
....

(3) $5x = 20$, where $x \in Q$

....
....
....
....

(4) $\frac{2}{5}x = \frac{1}{5}$, where $x \in Q$

....
....
....
....

(5) $-4 + y = 15$, where $x \in N$

....
....
....
....

(6) $m - (-3) = 1$, where $x \in Z$

....
....
....
....

[2] Find the solution set of each of the following equations:

(1) $2x - 1 = 5$, where $x \in Q$

....
....
....
....
....
....

(2) $8x + 4 = 12$, where $x \in Q$

....
....
....
....
....
....

(3) $3x - 13 = 26$, where $x \in N$

....
....
....
....
....
....

(4) $8 + 2x = 14$, where $x \in Z$

....
....
....
....
....
....

(5) $8 - 2x = -2$, where $x \in Z$

....
....
....
....
....
....

(6) $2x - 8 = -2$, where $x \in Z$

....
....
....
....
....
....

[3] Find the solution set of each of the following equations in Q:

(1) $2(x-3)=4$

……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………

(2) $3(x+2)+7(x-1)=12$

……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………

(3) $4(x-1)-(x+3)=0$

……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………

(4) $x+3=18-3x$

……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………

(5) $5x-4=2x+11$

……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………

(6) $7x-4=-2x+11$

……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………
……………………………………………

[4] Complete:

(1) If $x + 5 = 7$, then $x =$ ………

(2) If $3x = 6$, then the value of $6x =$ ………

(3) If $x + 9 = 11$, then the value of $7x =$ ………

(4) If $2y + 3 = 15$, then the value of $\frac{1}{2}y =$ ………

(5) If $2x = 2$, then $3x - 1 =$ …………

(6) If $2x = 0$, then $x =$ ………

(7) If the age of a man now is x years, then his age 5 years ago is ……

(8) If the age of a man now is y years, then his age after 4 years is ……

(9) If the age of a man after 5 years is x years, then his age now is ……

Life Applications

☞ Perimeter of rectangle = (L + W) × 2
If the example state perimeter of rectangle, we get half the perimeter and say: L + W = $\frac{1}{2}$ the perimeter.

☞ If the example state three consecutive integers, we suppose that the three numbers are: x , $x + 1$, $x + 2$

☞ If the example state three consecutive odd (or even) numbers, we suppose that the three numbers are: x , $x + 2$, $x + 4$

(1) The length of a rectangle exceeds its width by 4 meters and its perimeter is 68 meters. Find the dimensions of the rectangle.

....
....
....
....
....
....
....
....

(2) Find the numbers that if it is added to its triple the result is 32.

....
....
....
....
....
....
....
....

(3) Find the number which if we subtract 9 from its triple, the result will be 6.

....

....

....

....

....

....

....

....

(4) The sum of two consecutive numbers is 97. Find the two numbers

....

....

....

....

....

....

....

....

(5) Three consecutive natural numbers whose sum is 213 what are these numbers?

....

....

....

....

....

....

....

....

(6) The sum of three consecutive even numbers is 966. Find them.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

(7) Find three consecutive odd numbers if their sum is 357.

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

……………………………………………………………………………………………………

Lesson (8)

Solving inequalities in Q

[1] Find the solution set of each of the following inequalities in Q:

(1) $3x - 2 < 1$

....
....
....
....
....
....

(2) $4x + 2 \geq -10$

....
....
....
....
....
....

(3) $2x + 1 \leq 9$

....
....
....
....
....
....

(4) $-4x \geq -8$

....
....
....
....
....
....

(5) $3 - 2x \geq 1$

....
....
....
....
....
....

(6) $2 - 3x \leq 4$

....
....
....
....
....
....

(7) $3x - 1 \geq 2x + 3$

....
....
....
....
....
....

(8) $3x - 2 > x + 4$

....
....
....
....
....
....

(9) $3(x + 2) < -x + 4$

....
....
....
....
....
....

(10) $2(x+1) < x+4$

....
....
....
....
....
....

[2] Choose the correct answer:

(1) If $-x < 5$, then ....

(a) $x > 5$ (b) $x > -5$ (c) $x < 5$ (d) $x < -5$

(2) If $x \in N$, then the S.S. of the inequality $-x > 3$ is ....

(a) {4, 5, ...} (b) {-4, -5, ...} (c) {-3} (d) $\varnothing$

(3) $\frac{x}{3} < 4$ is equivalent to ....

(a) $x > \frac{4}{3}$ (b) $x < \frac{4}{3}$ (c) $x > 12$ (d) $x < 12$

(4) If $x \in Z$, then the S.S. of the inequality $20 < 5x < 25$ is ....

(a) {4} (b) {5} (c) {4, 5} (d) $\varnothing$

(5) The S.S. of the inequality $-2x < zero$ in Q is ....

(a) ∅ (b) Q_+ (c) Q_- (d) Z_+

(6) If $x > y$, then $\frac{1}{x}$......$\frac{1}{y}$ where $x \neq 0$ and $y \neq 0$

(a) < (b) > (c) = (d) ≥

(7) If $x > 5$, then $-x$ ....

(a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

Lesson (9)
Probability

☞ Suppose that

A is an any event, $n(A)$ is a number of elements of A and $n(S)$ is a number of elements of the sample space, then $P(A) = \frac{n(A)}{n(S)}$

☞ Coin:

Probability of getting a head = $\frac{1}{2}$ = 0.5 = 50%

Probability of getting a tail = $\frac{1}{2}$ = 0.5 = 50%

☞ Die:

As throwing a fair die once and observing the upper face, complete the following:

(1) The probability of appearance a number greater than 3 = ……

(2) The probability of appearance a number less than 3 = ……

(3) The probability of appearance an even number = ……

(4) The probability of appearance the number 4 = ……

(5) The probability of appearance the number 7 = ……

(6) The probability of appearance a number ≤ 6 = ……

(7) The probability of appearance a prime number = ……

[1] Complete:

(1) The probability of occurring the impossible event = ………… and the certain event = …………

(2) If a coin is flipped once, then the probability of appearance of head = ………

(3) ١٠ cards numbered from ١ to ١٠. If a card is drawn randomly, then the probability that the card in numbered by an odd number is ………

(4) A box has 5 white balls, 7 red balls and 3 blue balls. If a ball is drawn randomly from the box, then the probability that the ball is blue = …………

(5) In the experiment of throwing a fair die once and observing the upper face, the probability that the appearance number is less than 1 = …………

(6) A box contains 48 oranges, 4 of them are bad. If an orange is drawn randomly, then the probability that the drawn orange is bad = ……… and the probability of the drawn orange is good = ………

(7) If the probability of occurring an event is $\frac{5}{8}$, then the probability that the event doesn't occur = ………

(8) If the probability that a person get infected (in a city whose number of inhabitants is 200 000) with a disease is 0.003, then the expected number of infected persons with the disease in this city is …………………………… persons.

[2] Choose the correct answer:

(1) Which of the following may be a probability of an event?
(a) -0.25 (b) 87% (c) 1.05 (d) 130%

(2) A basket contains cards numbered from 1 to 20. If a card drawn randomly, what is the probability that the number written on it is divisible by 6?
(a) $\frac{3}{20}$ (b) $\frac{4}{20}$ (c) $\frac{5}{20}$ (d) $\frac{6}{20}$

[3] A box contains 10 balls numbered from 1 to 10, Find the prob. of:

(1) Number divisible by 7 = ……………………………………

(2) Number is an even = ……………………………………

(3) Number less than 8 = ……………………………………

(4) Odd greater than 3 = ……………………………………

(5) Prime number = ……………………………………………

(6) Number divisible by 5 = ……………………………………

[4] A school has 480 students, and the number of girls is 300, if a student is chosen randomly, then find the probability that the student is:

(1) A girl = ………………

(2) A boy = ………………

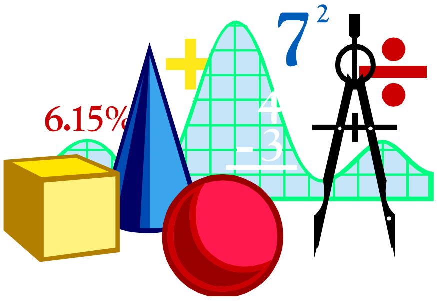
GEOMETRY

SECOND TERM

PREPARATORY ONE

PREPARED BY

Mr. MAHMOUD



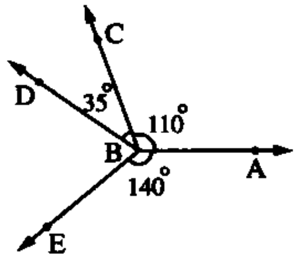
Lesson (1)
Deductive Proof

١. **In the opposite figure :**

m (∠ ABC) = 110° , m (∠ CBD) = 35° and

m (∠ ABE) = 140°

Find : m (∠ EBD)

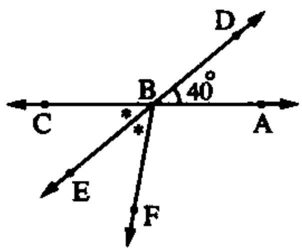


٢. **In the opposite figure :**

$\overleftrightarrow{AC} \cap \overleftrightarrow{DE} = \{B\}$, m (∠ ABD) = 40° and

$\overrightarrow{BE}$ bisects ∠ CBF

Find : m (∠ ABF)



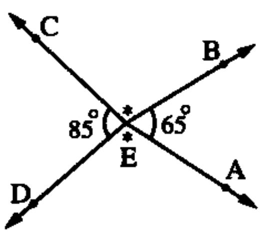
٣. **In the opposite figure :**

$\overrightarrow{EA} \cap \overrightarrow{EB} \cap \overrightarrow{EC} \cap \overrightarrow{ED} = \{E\}$,

If m (∠ BEC) = m (∠ AED)

Find : m (∠ BEC)

Are A , E , and C on the same straight line ? Why ?



In the opposite figure :

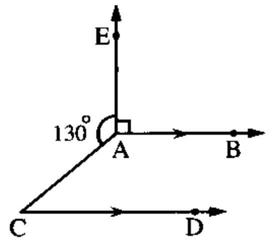
$\overrightarrow{AB} \parallel \overrightarrow{CD}$,

, m (∠ EAC) = 130° and

m (∠ EAB) = 90°

Find : (1) m (∠ BAC)

(2) m (∠ C)



« 140° , 40° »

٤. ....

....

....

....

....

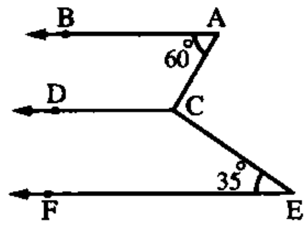
....

In the opposite figure :

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overrightarrow{AB} \parallel \overrightarrow{EF}$

, m (∠ A) = 60° and m (∠ E) = 35°

Find : m (∠ ACE)



٥. ....

....

....

....

....

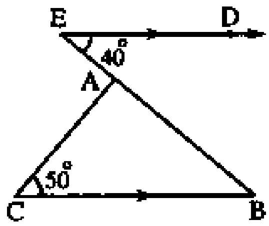
....

In the opposite figure :

$\overrightarrow{ED} \parallel \overrightarrow{CB}$, m (∠ C) = 50°

, m (∠ E) = 40°

Prove that : $\overleftrightarrow{AC} \perp \overleftrightarrow{BE}$



٦. ....

....

....

....

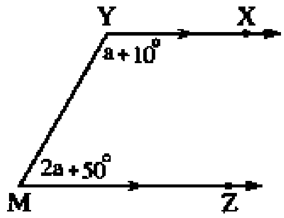
....

....

٧. **In the opposite figure :**

If $\overline{XY} \parallel \overline{MZ}$

Find the value of a

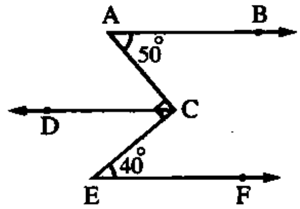


٨. **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, m ($\angle$ A) = 50° ,

$\angle$ ACE is right and m ($\angle$ E) = 40°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{EF}$



٩. **In the opposite figure :**

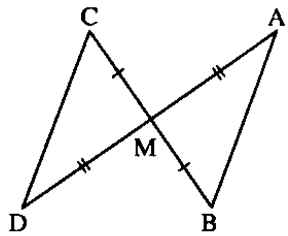
$\overline{AD} \cap \overline{BC} = \{M\}$,

MA = MD and MB = MC

Prove that :

(1) AB = CD

(2) $\overline{AB} \parallel \overline{CD}$



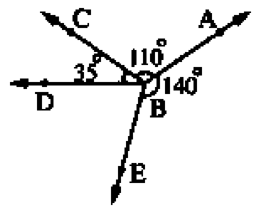
Homework

١٠. **In the opposite figure :**

m (∠ ABC) = 110° , m (∠ CBD) = 35° ,

m (∠ ABE) = 140° ,

Find : m (∠ EBD)



....

....

....

....

....

....

....

....

....

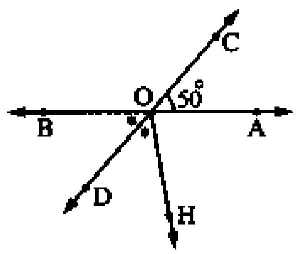
١١. **In the opposite figure :**

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{O\}$,

m (∠ AOC) = 50° ,

$\overrightarrow{OD}$ bisects ∠ HOB

Find : m (∠ AOH)



....

....

....

....

....

....

....

....

....

....

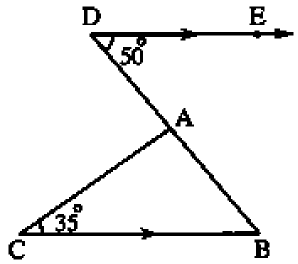
In the opposite figure :

$\overrightarrow{DE} \parallel \overrightarrow{CB}$, m (∠ D) = 50°

, m (∠ C) = 35°

Find : (1) m (∠ B)

(2) m (∠ BAC)



١٢.

....

....

....

....

....

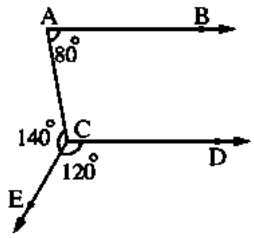
....

In the opposite figure :

m (∠ BAC) = 80° , m (∠ DCE) = 120°

and m (∠ ACE) = 140°

Prove that : $\overrightarrow{AB} \parallel \overrightarrow{CD}$



١٣.

....

....

....

....

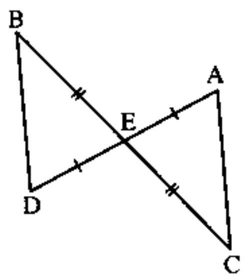
....

....

In the opposite figure :

$\overline{AD} \cap \overline{BC} = \{E\}$ where AE = DE and BE = CE

Prove that : Δ AEC ≡ Δ DEB



١٤.

....

....

....

....

....

....

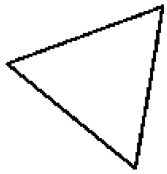
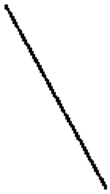
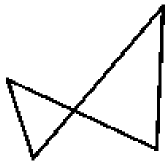
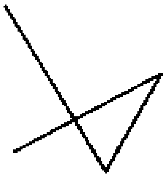
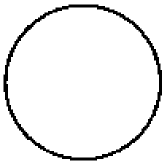
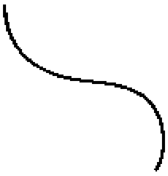
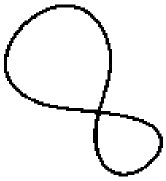
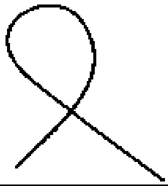
Lesson (2)
The Polygon

The simple line : It is the line that does not cut itself.

The non-simple line : It is the line that cuts itself once or more.

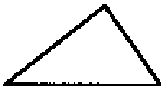
The closed line : It is the line that ends where it starts at the same point.
It may be simple or non-simple.

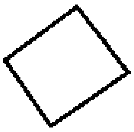
The open line : It is the line whose starting point is not the end point.
It may be simple or non-simple.

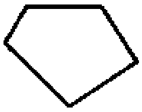
Simple line		Non-simple line	
Closed	**Open**	**Closed**	**Open**
			
			

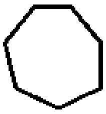
The polygon

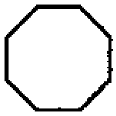
It is a simple closed line that consists of three line segments , or more. The polygon is named according to the number of its sides.


Triangle (3 sides)


Quadrilateral (4 sides)


Pentagon (5 sides)


Heptagon (7 sides)


Octagon (8 sides)

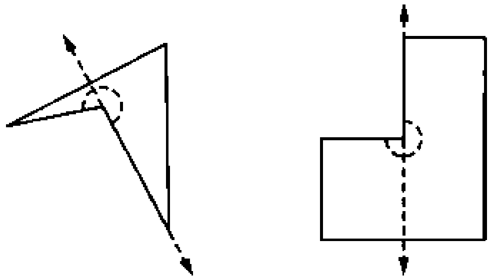
In the convex polygon :
If a straight line is drawn to pass through any two consecutive vertices, then the remained vertices lie on one side of this straight line as shown in the two figures.

Notice that :

Any interior angle of the convex polygon has measure less than 180°

In the concave polygon :

There are straight lines (one at least) passing through two consecutive vertices and the remained vertices lie on two different sides of the straight line as shown in the two opposite figures.



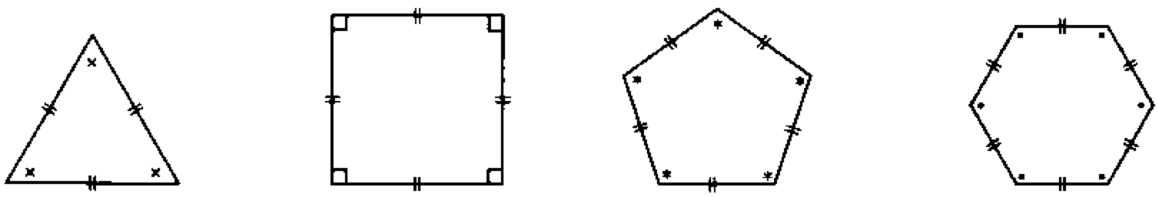
Notice that :

There is at least one interior angle of concave polygon of measure more than 180° (reflex angle).

The polygon is regular if :

1. All its sides are equal in length.
2. All its angles are equal in measure.

As examples for the regular polygons :



Equilateral triangle — Square — Regular pentagon — Regular hexagon

∴ **The sum of measures of the interior angles of a polygon of n sides equals (n – 2) × 180°**

So we get :

The sum of measures of the exterior angles of a convex polygon of n sides = 360°
(taking into account one exterior angle at each vertex)

∴ **The measure of each interior angle of the regular polygon of n-sides** $= \frac{(n-2)\times 180^\circ}{n}$

Remark

The number of sides of the regular polygon in which the measure of one of its interior angles is $x^\circ = \frac{360^\circ}{180^\circ - x}$

$No\ Of\ Diagonals = \frac{n(n-3)}{2}$

The polygon	No. Of its sides (n)	No. of its diagonals $\frac{n(n-3)}{2}$	The sum of measures of its interior angles $(n-2)\times 180$	The measure of the interior angle $\frac{(n-2)\times 180}{n}$
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
Heptagon				
Octagon				

Complete each of the following :

١.	The regular polygon is the one in which : (a) .... (b) ....
٢.	The sum of measures of the interior angles of the quadrilateral = ... °
٣.	The sum of measures of the interior angles of the pentagon = ...°
٤.	The sum of measures of the interior angles of the hexagon = ...°
٥.	The sum of measures of the interior angles of the heptagon = ...°
٦.	The measure of the interior angle of the regular pentagon = ... ° and the measure of the interior angle of the regular heptagon = ... °
٧.	The sum of measures of the exterior angles of the hexagon equals .... °

٨.	If the perimeter of a regular hexagon is 30 cm. , then its side length = ... cm. an the measure of each interior angle in it = ...
٩.	If the perimeter of a regular polygon = 80 cm. and its side length = 10 cm. , then the measure of each interior angle in it = ... °

Choose the correct answer :

١.	The sum of measures of the interior angles of a polygon of n sides = ... (a) $n \times 180°$ (b) $(n-2) \times 180°$ (c) $\frac{(n-2) \times 180°}{2}$ (d) $\frac{(n-2) \times 180°}{2n}$
٢.	The measure of the interior angle of a regular polygon of n sides equals .... (a) $\frac{(n-2) \times 90°}{n}$ (b) $\frac{(n-2) \times 180°}{2}$ (c) $\frac{(n-2) \times 180°}{n}$ (d) $180° \times (n-1)$
٣.	The measure of the interior angle of the regular polygon of 10 sides equals .... (a) 72° (b) 108° (c) 144° (d) 150°
٤.	The measure of the interior angle of a regular polygon of 18 sides equals .... (a) 130° (b) 140° (c) 150° (d) 160°
٥.	If the measure of an interior angle of a regular polygon is 135° , then the number of its sides is .... (a) 6 (b) 4 (c) 7 (d) 8
٦.	The sum of measures of the exterior angles of the triangle equals .... (a) 90° (b) 180° (c) 360° (d) 720°
٧.	In the quadrilateral ABCD , if $m(\angle A) = 2\, m(\angle B) = m(\angle C) = 96°$, then $m(\angle D) =$ .... (a) 96° (b) 48° (c) 120° (d) 144°

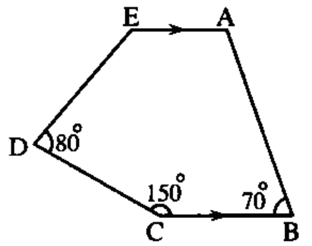
Essay problems:

In the opposite figure :

$\overline{AE} \parallel \overline{BC}$, m (∠ B) = 70° , m (∠ C) = 150° and

m (∠ D) = 80°

find m (∠ E)



١. ....

....

....

....

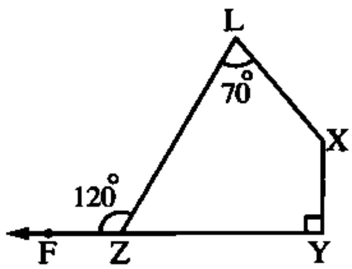
....

In the opposite figure :

F ∈ $\overrightarrow{YZ}$, m (∠ L) = 70° ,

m (∠ Y) = 90° and m (∠ LZF) = 120°

Find : m (∠ X)



٢. ....

....

....

....

....

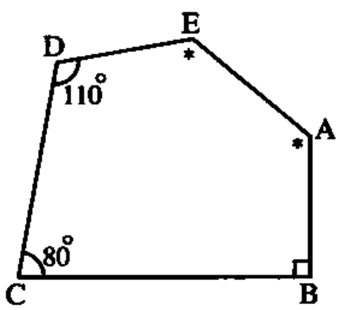
In the opposite figure :

If $\overline{AB} \perp \overline{BC}$, m (∠ C) = 80° ,

m (∠ D) = 110° and

m (∠ A) = m (∠ E)

٣. **Find :** m (∠ A)



....

....

....

....

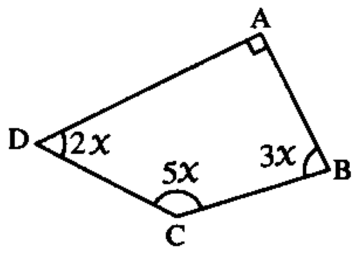
....

٤. **In the opposite figure :**

ABCD is a quadrilateral

in which : m (∠ A) = 90°

Find : The value of x



....

....

....

....

....

....

....

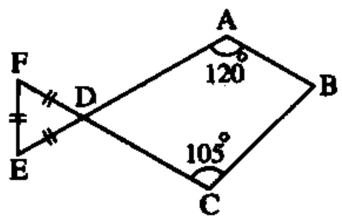
٥. **In the opposite figure :**

$\overline{AE} \cap \overline{CF} = \{D\}$,

Δ DEF is an equilateral triangle ,

m (∠ A) = 120° and m (∠ C) = 105°

Find : m (∠ B)



....

....

....

....

....

....

....

Homework

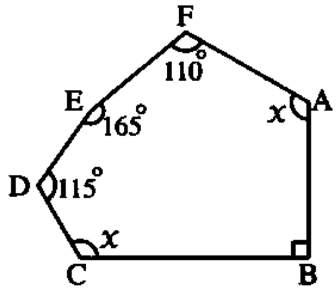
٦. In the opposite figure :

ABCDEF is a hexagon.

m (∠ FAB) = m (∠ DCB)

Find :

The value of x



....

....

....

....

....

....

....

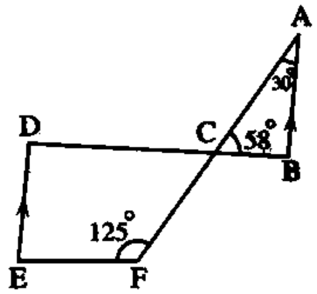
٧. **In the opposite figure :**

$\overline{BD} \cap \overline{AF} = \{C\}$, $\overline{AB} \parallel \overline{ED}$,

m (∠ A) = 30° and m (∠ ACB) = 58° ,

m (∠ CFE) = 125°

Find : m (∠ E)



....

....

....

....

....

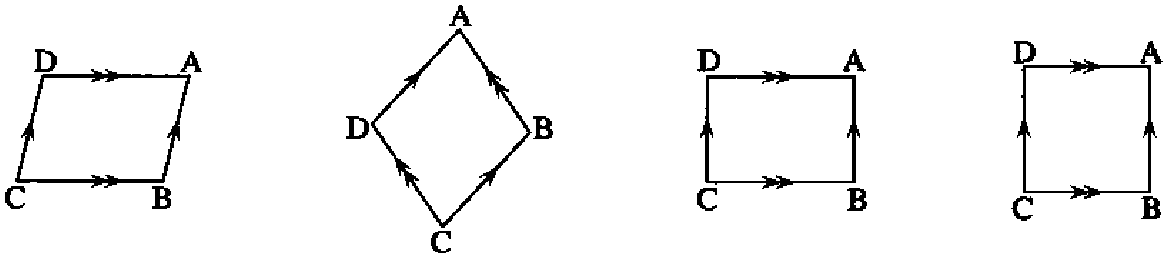
....

....

Lesson (3)
The parallelogram and its properties

Definition :

A parallelogram is a quadrilateral , in which each two opposite sides are parallel.

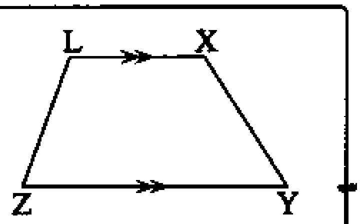


Each of the above figures is called a parallelogram for $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$

Notice that :

A quadrilateral in which only two sides are parallel is called a trapezium , as shown in the opposite figure in which :

$\overline{XL} \parallel \overline{YZ}$



When does a quadrilateral represent a parallelogram ?

A quadrilateral represents a parallelogram if <u>one</u> of the following conditions satisfies

Each two opposite sides are parallel.	Each two opposite sides are equal in length.	Two opposite sides are parallel and equal in length.	Each two opposite angles are equal in measure	The two diagonals bisect each other.

Complete each of the following :

١. In a parallelogram , every two opposite sides are .... , ...

٢. In a parallelogram , every two opposite angles are ....

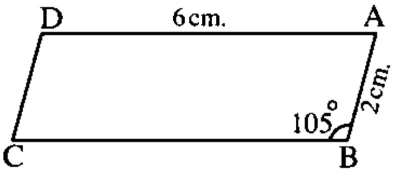
٣. In a parallelogram , every two consecutive angles are ....

٤. In a parallelogram , the two diagonals ....

٥. The quadrilateral in which two sides are parallel is called ....

٦. A quadrilateral represents a parallelogram if ....

٧.	ABCD is a parallelogram in which m (∠ A) = 50° , then m (∠ B) = ………. °
٨.	In the parallelogram XYZL , if m (∠ X) = $\frac{1}{2}$ m (∠ Y) , then m (∠ Y) = ………. °
٩.	**In the opposite figure :** ABCD is a parallelogram in which AB = 2 cm. , AD = 6 cm. and m (∠ B) = 105° **Complete the following :** (1) BC = ………. cm. , DC = ………. cm. (2) m (∠ D) = ……….° , m (∠ A) = ……….° and m (∠ C) = ……….° (3) The perimeter of the parallelogram ABCD = ………. cm.



Choose the correct answer :

١.	ABCD is a parallelogram in which : m (∠ A) = 50° , then m (∠ C) = ………. (a) 50° (b) 60° (c) 130° (d) 150°
٢.	ABCD is a parallelogram in which : m (∠ A) + m (∠ C) = 140° , then m (∠ B) = ………. (a) 70° (b) 40° (c) 110° (d) 220°
٣.	If the lengths of two consecutive sides of a parallelogram are 3 cm. and 5 cm. , then its perimeter equals ………. cm. (a) 12 (b) 14 (c) 16 (d) 18
٤.	If the perimeter of a parallelogram is 25 cm. and if one of its sides is of length 7 cm. , then the consecutive side is of length ………. cm. (a) 7 (b) 18 (c) 12.5 (d) 5.5

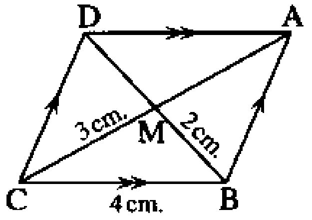
Essay problems:

١. **In the opposite figure :**

ABCD is a parallelogram whose diagonals intersect at M

If BC = 4 cm. , BM = 2 cm. and MC = 3 cm. ,

then find the perimeter of Δ AMD



....

....

....

....

....

....

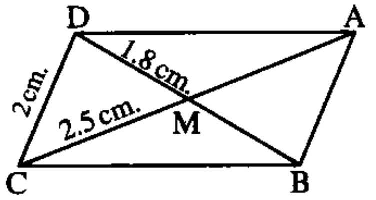
٢. **In the opposite figure :**

ABCD is a parallelogram such that :

$\overline{AC} \cap \overline{BD} = \{M\}$ If CD = 2 cm. ,

MC = 2.5 cm. and MD = 1.8 cm.

Calculate the perimeter of Δ AMB



« 6.3 cm. »

....

....

....

....

....

....

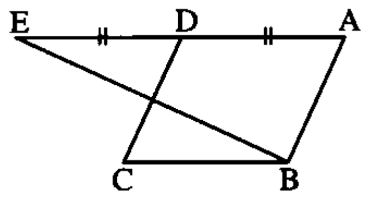
....

٣. **In the opposite figure :**

ABCD is a parallelogram,

$E \in \overrightarrow{AD}$ in which : AD = DE

Prove that : $\overline{DC}$ and $\overline{BE}$ bisect each other.



....

....

....

....

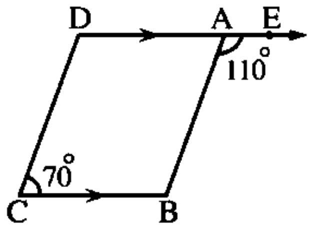
....

....

....

٤.

In the opposite figure :



ABCD is a quadrilateral in which :

$\overline{AD} \parallel \overline{BC}$, $E \in \overrightarrow{DA}$, m (∠ BAE) = 110°

and m (∠ DCB) = 70°

Prove that : ABCD is a parallelogram

....

....

....

....

....

....

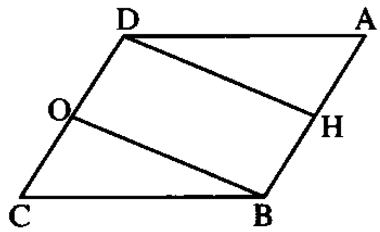
....

....

....

٥.

In the opposite figure :



ABCD is a parallelogram ,

H is the midpoint of $\overline{AB}$

and O is the midpoint of $\overline{DC}$

Prove that : HBOD is a parallelogram.

....

....

....

....

....

....

....

....

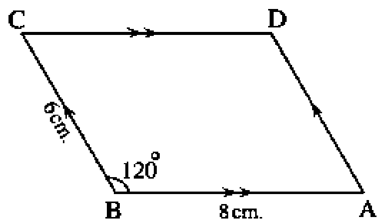
....

Homework

6. **In the opposite figure :**

ABCD is a parallelogram in which :

AB = 8 cm. , BC = 6 cm. and m (∠ B) = 120°



Find : **1** The length of each of $\overline{CD}$ and $\overline{DA}$

2 The measure of each of ∠ D , ∠ A and ∠ C

3 The perimeter of ABCD

....

....

....

....

....

....

....

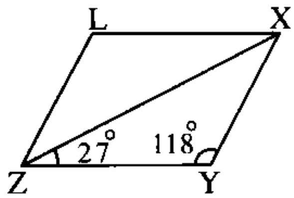
....

....

7. **In the opposite figure :**

XYZL is a parallelogram in which :

m (∠ Y) = 118° , m (∠ XZY) = 27°



Find :

(1) m (∠ YXZ) | (2) m (∠ LZX)

(3) m (∠ LXZ) | (4) m (∠ L)

« 35° , 35° , 27° , 118° »

....

....

....

....

....

....

....

....

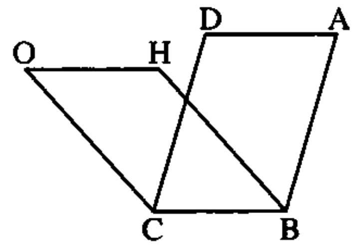
....

٨. **In the opposite figure :**

Each of ABCD

and HBCO is a parallelogram

Prove that : AD = HO

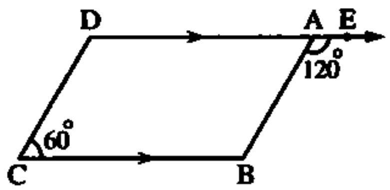


٩. **In the opposite figure :**

$E \in \overrightarrow{DA}$, m (∠ EAB) = 120°

m (∠ C) = 60° , $\overrightarrow{DA} \,//\, \overline{CB}$

Prove that : ABCD is a parallelogram.



١٠. **In the opposite figure :**

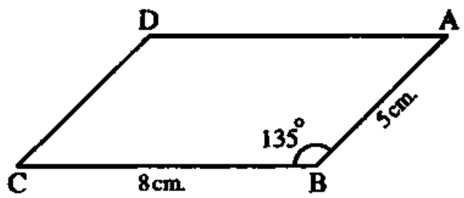
ABCD is a parallelogram in which

AB = 5 cm. , BC = 8 cm. , m (∠ B) = 135°

Find :

(1) m (∠ C)

(2) The perimeter of parallelogram ABCD



Lesson (4)
The special cases of the parallelogram

Notice that :

To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram , as we see in the previous lesson , then :

The parallelogram is

a rectangle	a rhombus	a square
if : One of its angles is a right angle **or** Its two diagonals are equal in length.	**if :** Two adjacent sides are equal in length. **or** Its two diagonals are perpendicular.	**if :** One of its angles is right and two adjacent sides are equal in length. **or** One of its angles is right and its two diagonals are perpendicular. **or** Its two diagonals are perpendicular and equal in length. **or** Two adjacent sides are equal in length and its two diagonals are equal in length.

Complete each of the following :

١.	A parallelogram whose two diagonals are perpendicular is called ....
٢.	The parallelogram whose two diagonals are .... is called a rectangle.
٣.	The parallelogram whose two diagonals are equal in length and perpendicular is called ....
٤.	The quadrilateral whose sides are equal in length is called ....
٥.	The quadrilateral whose diagonals bisect each other is called ....
٦.	The rectangle is a .... with a right angle.

٧. The rhombus is a ………. in which its diagonals are perpendicular.

٨. The square is a ………. with a right angle.

٩. The rhombus whose two diagonals are equal in length is called ……….

١٠. The rectangle in which its two diagonals are perpendicular is called ……….

١١. The rectangle in which its two adjacent sides have the same length is called ……….

١٢. If $\overline{XY} \parallel \overline{ZL}$, XY = ZL , then the quadrilateral XYZL is called ……….

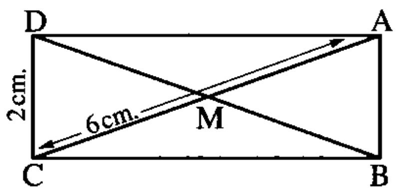
١٣. If ABCD is a rhombus , then ………. $\perp$ ……….

١٤. The perimeter of the square = ………. ,
The perimeter of the rectangle = ………. and
The perimeter of the rhombus = ……….

١٥. The rhombus whose perimeter is 42 cm. , its side length = ………. cm.

١٦. **In the opposite figure :**

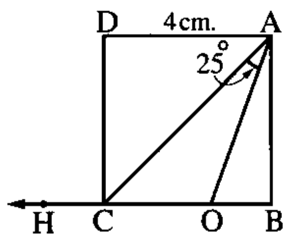
ABCD is a rectangle in which :
AC = 6 cm. , CD = 2 cm.
and $\overline{AC} \cap \overline{BD} = \{M\}$



Complete : (1) AB = ………. cm.
(2) DM = ………. cm.
(3) The perimeter of Δ ABM = ………. cm.

١٧. **In the opposite figure :**

ABCD is a square in which AD = 4 cm. ,
O $\in \overline{BC}$ such that : m ($\angle$ OAC) = 25°
and H $\in \overrightarrow{BC}$



Complete the following :
(1) The perimeter of the square = ………. cm.
(2) m ($\angle$ ACH) = ……….°
(3) m ($\angle$ AOC) = ……….°

Choose the correct answer :

١.	The two diagonals of a rectangle .... (a) are perpendicular. (b) are equal in length. (c) are perpendicular and equal in length. (d) bisect its interior angles.
٢.	The two diagonals of a rhombus are .... (a) perpendicular and are not equal. (b) equal in length and are not perpendicular. (c) perpendicular and equal in length. (d) not equal in length and are not perpendicular.
٣.	The two diagonals of the square ، are .... (a) just perpendicular. (b) just equal in length. (c) perpendicular and equal in length. (d) not equal in length and are not perpendicular.
٤.	If two adjacent sides are equal in length in a parallelogram، then the figure is a .... (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
٥.	If : ABCD is a rectangle in which AC = 5 cm.، then : BD = ... cm. (a) 2.5 (b) 5 (c) 10 (d) 20
٦.	If : ABCD is a square، then : m (∠ CAB) = ... (a) 90° (b) 45° (c) 60° (d) 30°
٧.	If : ABCD is a parallelogram in which m (∠ A) = m (∠ B)، then : ABCD is a .... (a) rectangle. (b) rhombus. (c) square. (d) trapezium.
٨.	If : ABCD is a rhombus in which m (∠ ACB) = 32°، then : m (∠ D) = ... (a) 32° (b) 64° (c) 116° (d) 26°

Essay problems:

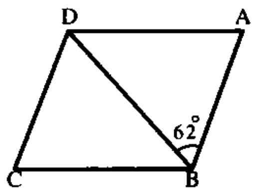
1. **In the opposite figure :**

ABCD is a rhombus,

$\overline{BD}$ is a diagonal in it ,

m (∠ ABD) = 62°

Find with proof : m (∠ A)



« 56° »

....

....

....

....

....

....

....

....

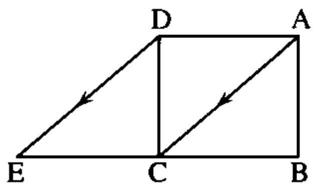
....

2. **In the opposite figure :**

ABCD is a square, E ∈ $\overrightarrow{BC}$, $\overline{AC}$ // $\overline{DE}$

Prove that : ACED is a parallelogram.

Find : m (∠ ACE)



« 135° »

....

....

....

....

....

....

....

....

....

Lesson (5)
The Triangle

Theorem (1) :

The sum of the measures of the interior angles of a triangle is 180°

The measure of the exterior angle of a triangle :

The measure of the exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.

Notice that :

The measure of the exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

Remark (1)

If two angles of one triangle equal two angles of another triangle in measure , then the third angle of the first triangle is equal in measure to the third angle of the other triangle.

Remark (2)

- If the sum of measures of two angles in a triangle equals 90° , then the third angle is right.
- If the sum of measures of two angles in a triangle is less than 90° , then the third angle is obtuse.
- If the sum of measures of two angles in a triangle is more than 90° , then the third angle is acute.

Remark (3)

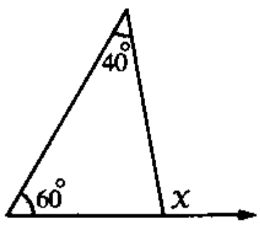
If the measure of an angle in a triangle equals the sum of measures of the other two angles , then the triangle is right-angled.

Complete each of the following :

١.	The sum of measures of the inteior angles of a triangle = ... °
٢.	The measure of the exterior angle of a triangle is equal to the sum of ....
٣.	If the measure of an angle in a triangle equals the sum of measures of the other two angles in the triangle , then the triangle is ....
٤.	In Δ ABC : If m (∠ A) + m (∠ C) = m (∠ B) , then m (∠ B) = ... °

٥.	If the measure of an angle in a triangle is greater than the sum of measures of the other two angles , then the triangle is ....
٦.	In Δ ABC : If m (∠ B) > m (∠ A) + m (∠ C) , then ∠ B is ....
٧.	It is possible to find a triangle each of its interior angles is of measure ....°

Choose the correct answer :

١.	The triangle contains two .... angles at least. (a) acute (b) obtuse (c) right (d) reflex
٢.	The sum of measures of the interior angles of a triangle equals the measure of .... angle. (a) a right (b) a straight (c) an acute (d) a reflex
٣.	In Δ XYZ, if : m (∠ X) = 50° , m (∠ Y) = 100°, then : m (∠ Z) = ... (a) 30° (b) 50° (c) 80° (d) 100°
٤.	In Δ ABC, if : m (∠ A) + m (∠ B) = 110°, then : m (∠ C) = ... (a) 110° (b) 90° (c) 70° (d) 55°
٥.	If the measures of two angles in a triangle are 35° and 45°, then the triangle is .... (a) acute-angled (b) right-angled (c) obtuse-angled (d) equilateral
٦.	The measure of the exterior angle of the equilateral triangle at any one of its vertices equals .... (a) 60° (b) 120° (c) 150° (d) 30°
٧.	**In the opposite figure :** x = ... (a) 60 – 40 (b) 60 × 40 (c) 60 + 40 (d) (60 ÷ 40) 
٨.	ABC is a triangle, m (∠ A) = $2x°$, m (∠ C) = $x°$ and m (∠ B) = $3x°$, then ∠ B is .... (a) acute. (b) right. (c) obtuse. (d) straight.

Essay problems:

In each of the following figures , find the measure of the angle marked by (?) :

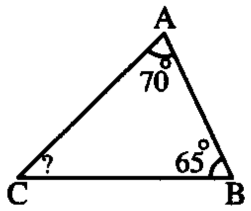


fig. (1)

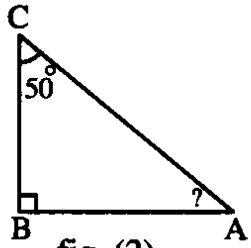


fig. (2)

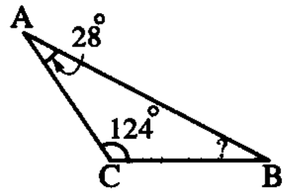


fig. (3)

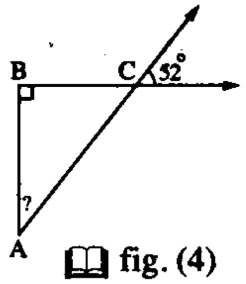


fig. (4)

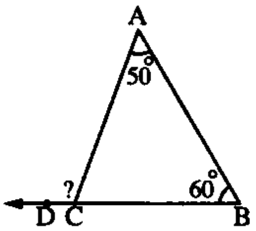


fig. (5)

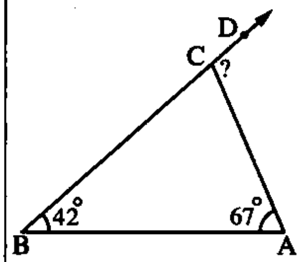


fig. (6)

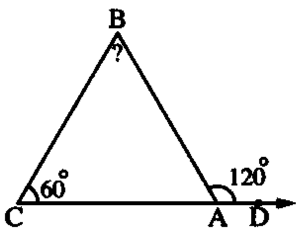


fig. (7)

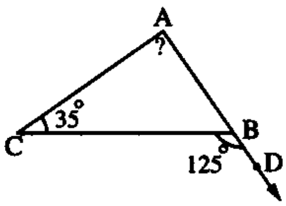


fig. (8)

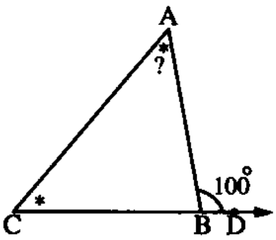


fig. (9)

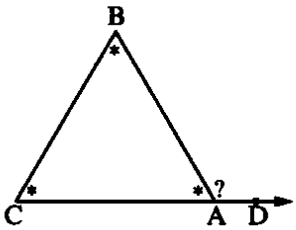


fig. (10)

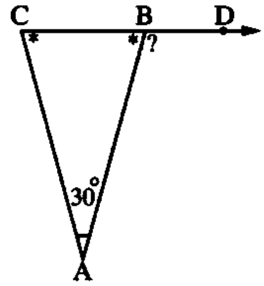


fig. (11)

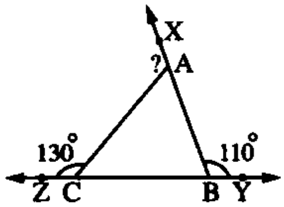


fig. (12)

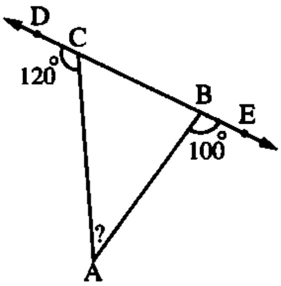


fig. (13)

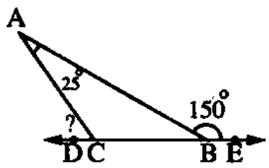


fig. (14)

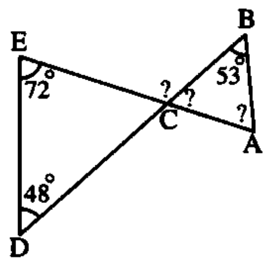


fig. (15)

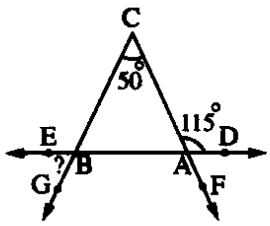


fig. (16)

٢.

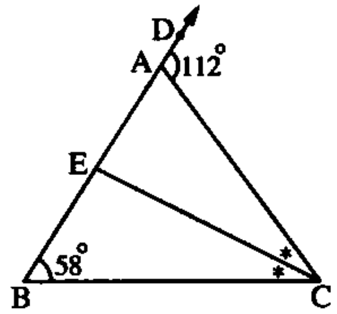
In the opposite figure :

ABC is a triangle in which : m (∠ B) = 58° ,

E ∈ $\overrightarrow{AB}$ such that $\overrightarrow{CE}$ bisects ∠ ACB ,

D ∈ $\overrightarrow{BA}$ and m (∠ CAD) = 112°

Find : m (∠ AEC)



....

....

....

....

....

....

....

....

....

٣.

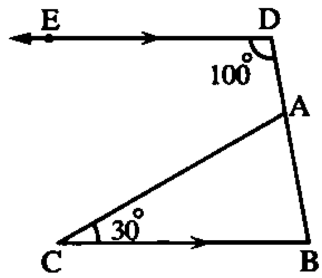
In the opposite figure :

$\overrightarrow{DE}$ // $\overrightarrow{BC}$, m (∠ D) = 100° ,

m (∠ C) = 30° and

A ∈ $\overline{DB}$

Find : m (∠ BAC)



....

....

....

....

....

....

....

....

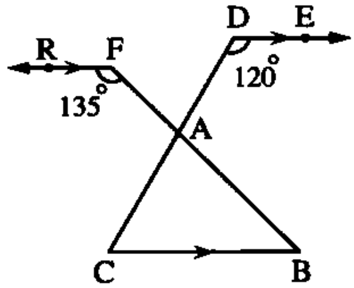
....

٤. **In the opposite figure :**

$\overrightarrow{DE}$ // $\overrightarrow{FR}$ // $\overline{BC}$,

m (∠ CDE) = 120° and m (∠ RFB) = 135°

Calculate the measures of the angles of Δ ABC



....

....

....

....

....

....

....

....

....

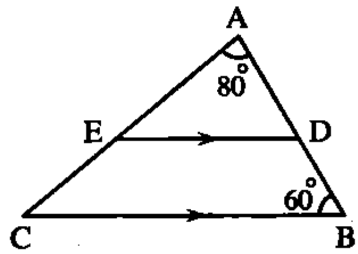
٥. **In the opposite figure :**

ABC is a triangle in which : m (∠ A) = 80° and

m (∠ B) = 60°

$\overline{DE}$ // $\overline{BC}$ where : D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$

Find : m (∠ AED) and m (∠ DEC)



....

....

....

....

....

....

....

....

....

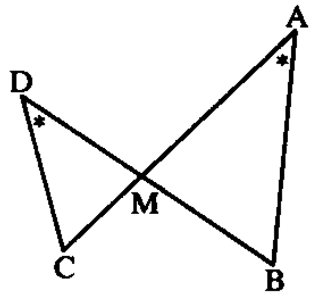
٦.

In the opposite figure :

$\overline{AC} \cap \overline{BD} = \{M\}$ and

m (∠ A) = m (∠ D)

Prove that : m (∠ B) = m (∠ C)



....

....

....

....

....

....

....

....

....

٧.

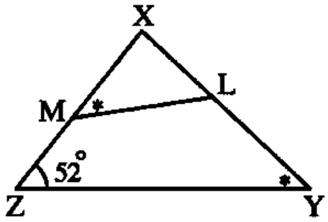
In the opposite figure :

XYZ is a triangle in which m (∠ Z) = 52° ,

L ∈ $\overline{XY}$ and M ∈ $\overline{XZ}$ such that :

m (∠ Y) = m (∠ XML)

Find : m (∠ XLM)



....

....

....

....

....

....

....

....

....

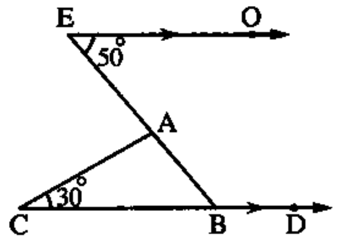
٨. **In the opposite figure :**

$\overrightarrow{EO} \parallel \overline{CD}$, m (∠ E) = 50°

, m (∠ C) = 30° ,

Find the measures of

angles of Δ ABC , m (∠ ABD)



....

....

....

....

....

....

....

....

....

Homework

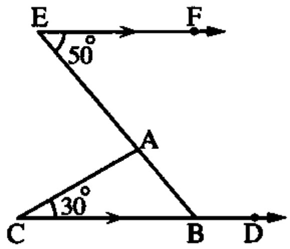
٩. **In the opposite figure :**

$\overrightarrow{EF} \parallel \overrightarrow{CD}$, m (∠ E) = 50° and

m (∠ C) = 30°

Find the measures of the angles

of Δ ABC and m (∠ ABD)



« m (∠ ABC) = 50° , m (∠ BAC) = 100° , m (∠ ABD) = 130° »

....

....

....

....

....

....

....

....

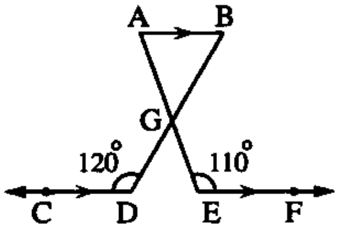
....

١٠. **In the opposite figure :**

$\overleftrightarrow{AB} \parallel \overleftrightarrow{DC} \parallel \overleftrightarrow{EF}$, $m(\angle E) = 110°$ and

$m(\angle D) = 120°$

Find : $m(\angle EGD)$



....

....

....

....

....

....

....

....

....

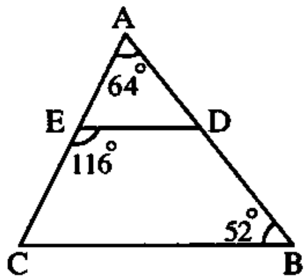
١١. **In the opposite figure :**

ABC is a triangle in which $m(\angle A) = 64°$,

$m(\angle B) = 52°$,

$m(\angle DEC) = 116°$, $E \in \overline{AC}$ and $D \in \overline{AB}$

Prove that : $\overline{DE} \parallel \overline{BC}$



....

....

....

....

....

....

....

....

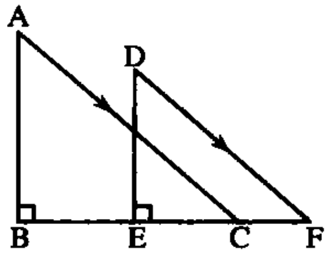
....

١٢. **In the opposite figure :**

The points F , C , E and B are collinear ,

m (∠ B) = m (∠ DEC) = 90° and $\overline{AC}$ // $\overline{DF}$

Prove that : m (∠ A) = m (∠ D)



....

....

....

....

....

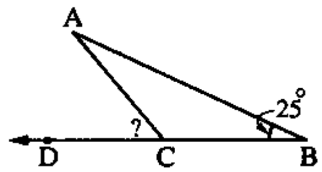
....

....

١٣. **In the opposite figure :**

m (∠ A) = m (∠ B) = 25°

Find : m (∠ ACD)



....

....

....

....

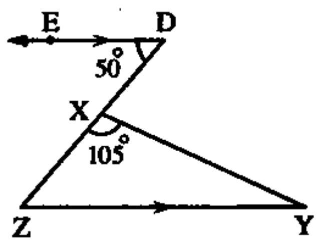
....

١٤. **In the opposite figure :**

$\overrightarrow{DE}$ // $\overline{YZ}$, m (∠ ZDE) = 50°

, m (∠ YXZ) = 105°

Find : m (∠ Z) , m (∠ Y) , m (∠ YXD)



....

....

....

....

....

....

....

Lesson (6)
Follow the triangle

Theorem (2) :

The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.

Corollary

The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

Theorem (3) :

The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

Complete each of the following :

١. The ray drawn from the midpoint of a side of a triangle parallel to another side ....

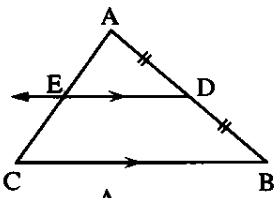
٢. The line segment joining the midpoints of two sides of a triangle is .... the third side.

٣. The length of the line segment joining the midpoints of two sides of a triangle equals ....

٤. **In the opposite figure :**

If D is the midpoint of $\overline{AB}$, $\overrightarrow{DE}$ // $\overline{BC}$

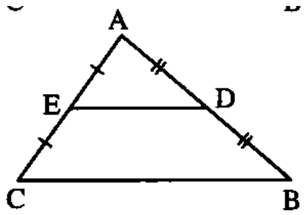
, then : .... is the midpoint of ....



٥. **In the opposite figure :**

If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, then : .... // ....

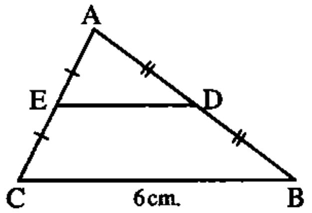


٦. **In the opposite figure :**

$\because$ D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, BC = 6 cm.

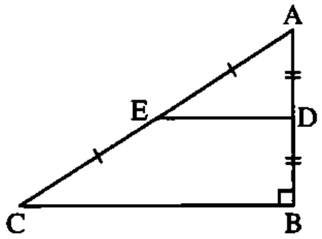
$\therefore$ DE = ·········· cm



٧. **In the opposite figure :**

If : m ($\angle$ B) = 90° , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, then : m ($\angle$ ADE) = ··········°

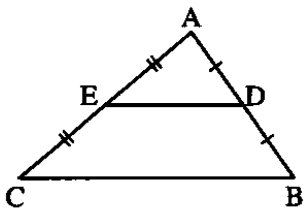


٨. **In the opposite figure :**

If : D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, and the perimeter of the triangle ABC = 24 cm.

, then the perimeter of the triangle ADE = ·········· cm.

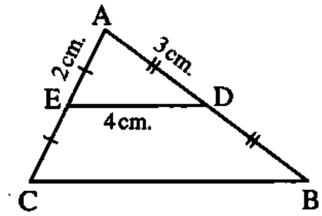


٩. **In the opposite figure :**

$\because$ D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, AD = 3 cm., AE = 2 cm. and DE = 4 cm.

$\therefore$ The perimeter of the figure DBCE = ·········· cm.

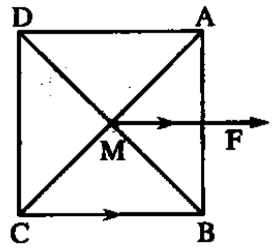


١٠. **In the opposite figure :**

If the perimeter of the square ABCD = 20 cm.

, $\overrightarrow{MF}$ // $\overline{CB}$ where F $\in$ $\overline{AB}$

, then : AF = ·········· cm.

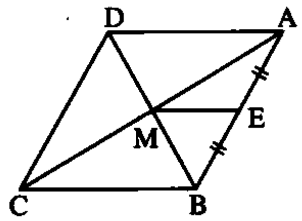


١١. **In the opposite figure :**

$\because$ The perimeter of the rhombus ABCD = 24 cm.,

E is the midpoint of $\overline{AB}$

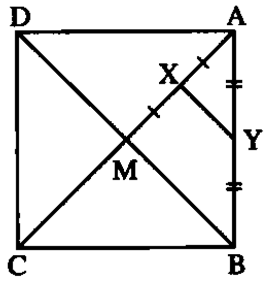
$\therefore$ ME = ·········· cm.



١٢. **In the opposite figure :**

∵ ABCD is a square , X and Y are the midpoints of $\overline{AM}$ and $\overline{AB}$ respectively and AC = 12 cm.

∴ XY = ... cm. , m (∠ AYX) = ...°

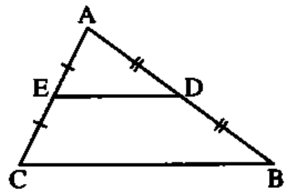


Choose the correct answer :

١. **In the opposite figure :**

If D , E are the midpoints if $\overline{AB}$, $\overline{AC}$ respectively and the perimeter if Δ AED = 24 cm. then the perimeter if Δ ABC = ...

(a) 12 (b) 18 (c) 48 (d) 24



Essay problems:

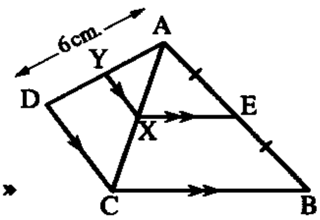
١. **In the opposite figure :**

AE = EB , $\overline{EX}$ // $\overline{BC}$, $\overline{XY}$ // $\overline{CD}$

and AD = 6 cm.

Find the length of : $\overline{AY}$

« 3 cm. »



....

....

....

....

....

....

....

....

....

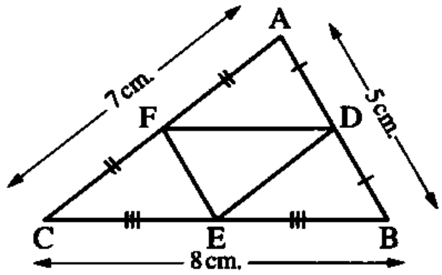
....

٢. **In the opposite figure :**

AB = 5 cm. , BC = 8 cm. ,

AC = 7 cm. , D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Calculate the perimeter of : Δ DEF



....

....

....

....

....

....

....

....

....

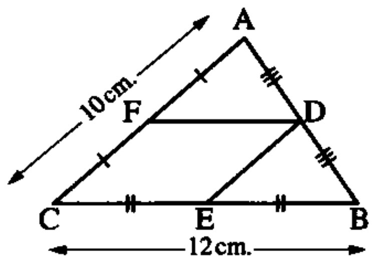
....

٣. **In the opposite figure :**

ABC is a triangle in which D , E and F are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively,

BC = 12 cm. , AC = 10 cm.

Find the perimeter of the figure DECF



....

....

....

....

....

....

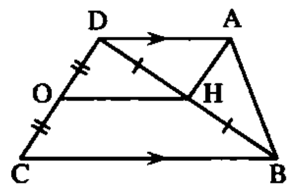
....

....

....

....

٤. **In the opposite figure :**



$\overline{AD} // \overline{BC}$, $AD = \frac{1}{2} BC$,

H is the midpoint of $\overline{BD}$,

O is the midpoint of $\overline{CD}$

Prove that : AHOD is a parallelogram.

....

....

....

....

....

....

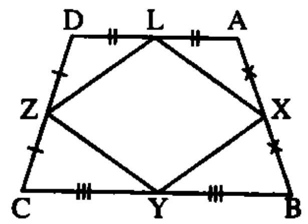
....

....

....

....

٥. **In the opposite figure :**



ABCD is a quadrilateral in which X , Y , Z and L

are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$ respectively.

Prove that : XYZL is a parallelogram.

....

....

....

....

....

....

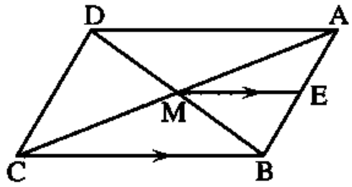
....

....

....

....

٦. **In the opposite figure :**



ABCD is a parallelogram , M is the intersection of its diagonals

Draw $\overrightarrow{ME}$ // $\overline{CB}$

is AE = EB ? giving reason.

....

....

....

....

....

....

....

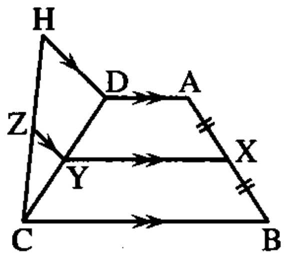
....

....

....

Homework

٧. **In the opposite figure :**



ABCD is a trapezium , $\overline{AD}$ // $\overline{BC}$, X is the midpoint of $\overline{AB}$

If $\overline{AD}$ // $\overline{XY}$ where Y $\in \overline{DC}$, $\overline{YZ}$ // $\overline{DH}$

Prove that : CZ = ZH

....

....

....

....

....

....

....

....

....

....

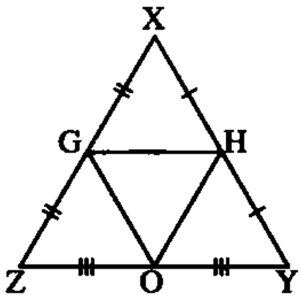
٨. **In the opposite figure :**

XYZ is a triangle in which :

H , O and G are the midpoints of $\overline{XY}$, $\overline{YZ}$ and $\overline{ZX}$ respectively.

If the perimeter of Δ HOG is 18 cm. ,

then find the perimeter of : Δ XYZ



....

....

....

....

....

....

....

....

....

....

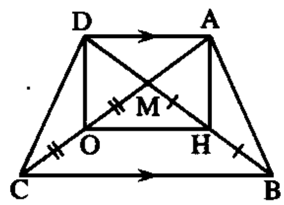
٩. **In the opposite figure :**

ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$

and AD = $\frac{1}{2}$ BC and $\overline{AC} \cap \overline{DB} = \{M\}$

Let H and O are the midpoints of $\overline{MB}$ and $\overline{MC}$ respectively.

Prove that : AHOD is a parallelogram.



....

....

....

....

....

....

....

....

....

....

....

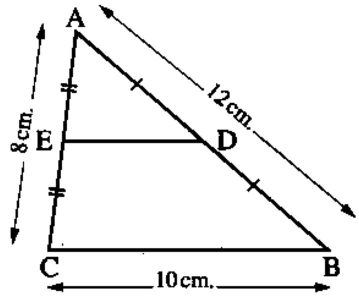
....

١٠.

In the opposite figure :

Δ ABC in which : AB = 12 cm. , BC = 10 cm. , AC = 8 cm.

Find the perimeter of : Δ ADE



....

....

....

....

....

....

....

....

....

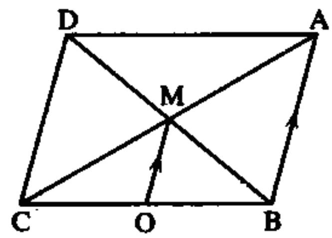
....

١١.

In the opposite figure :

ABCD is a parallelogram its diagonals are intersect at M , $\overrightarrow{MO} \parallel \overline{AB}$, $\overline{MO} \cap \overline{BC} = \{O\}$

Prove that : BO = OC



....

....

....

....

....

....

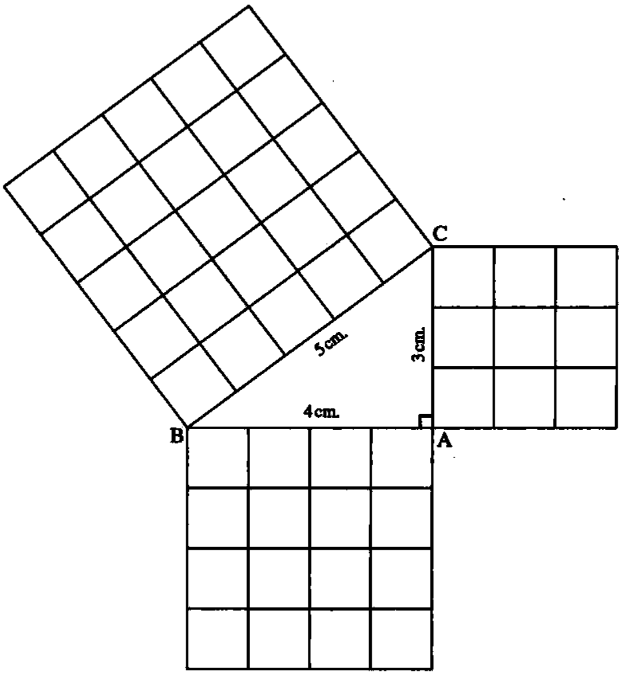
....

....

....

....

Lesson (7)
Pythagoras' theorem



Pythagoras' theorem :

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.

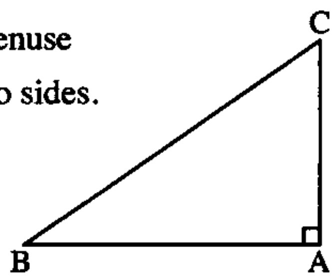
We can also write the previous theorem as follows :

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

i.e.

If ABC is a right-angled triangle at A, then :

$(BC)^2 = (AB)^2 + (AC)^2$



- **From the previous relation, we can deduce the following two relations :**

$(AB)^2 = (BC)^2 - (AC)^2$ $(AC)^2 = (BC)^2 - (AB)^2$

In each of the following figures , find the length of the unknown side :

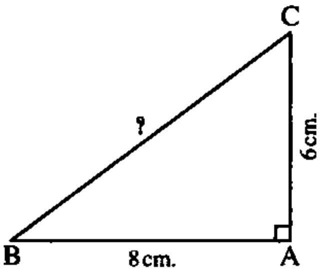


Fig. (1)

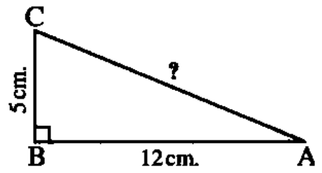


Fig. (2)

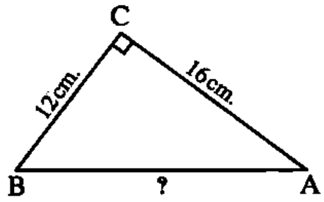


Fig. (3)

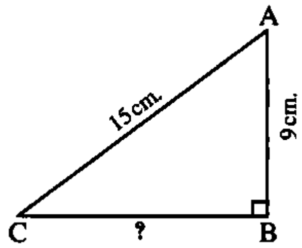


Fig. (4)

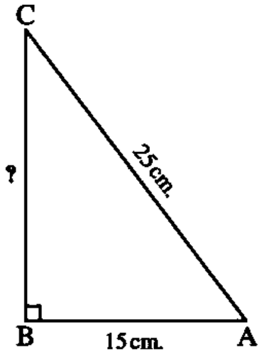


Fig. (5)

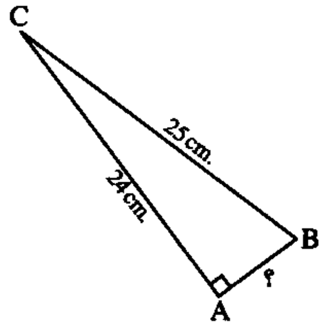


Fig. (6)

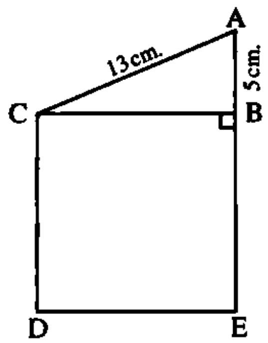
Complete each of the following :

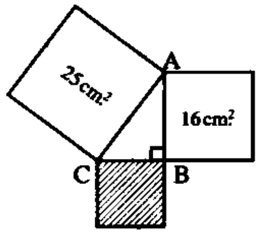
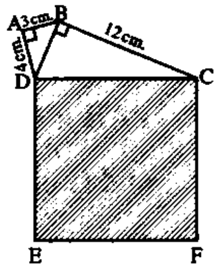
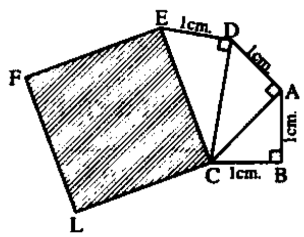
١. In the right-angled triangle , the area of the square on the hypotenuse equals ....

٢. If XYZ is a right-angled triangle at X , XY = 12 cm. and XZ = 9 cm. ,
then YZ = ... cm.

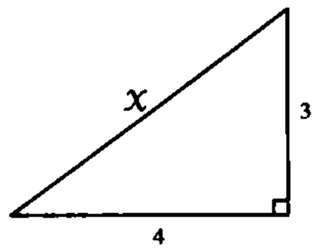
٣. If ABC is a right-angled triangle at B , AB = 20 cm. and AC = 25 cm. ,
then BC = ... cm.

٤. **In the opposite figure :**
If m (∠ ABC) = 90°
, AB = 5 cm.
and AC = 13 cm.
, then the area of
the square BEDC = ... cm^2.



٥.	A rectangle is of length 8 cm. and width 6 cm. , then the length of its diagonal equals ·········· cm.
٦.	If the area of a rectangle equals 60 cm^2 and its width is 5 cm. , then the length of its diagonal = ·········· cm.
٧.	its diagonal = ·········· cm. If Δ ABC is right-angled at B , then the side length of the shaded square = ·········· cm. 
٨.	If Δ ABD is right-angled at A and Δ BCD is right-angled at B , then the area of the shaded square = ·········· cm^2 
٩.	If each of the triangles ABC , ACD and DCE are right-angled at B , A and D respectively, AB = BC = AD = DE = 1 cm. , then the area of the shaded square = ·········· cm^2 
١٠.	If the lengths of the two diagonals of a rhombus are 6 cm. and 8 cm. , then its side length = ··········

Choose the correct answer :

١.	**In the opposite figure :** Which of the following relations is true ? (a) $x = 4^2 + 3^2$ (b) $x^2 = 4^2 - 3^3$ (c) $x^2 + 9 = 16$ (d) $x^2 = 25$ 
٢.	If ABCD is a square , then $(AC)^2$ = ·········· (a) AB (b) $(AB)^2$ (c) $2\ (AB)^2$ (d) $4\ (AB)^2$

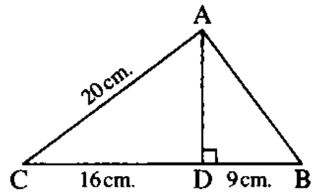
Essay problems:

١. **In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, BD = 9 cm. ,

DC = 16 cm. and AC = 20 cm.

Find :



(1) AD (2) AB

(3) The area of Δ ABC « 12 cm. , 15 cm. , 150 cm^2 »

....

....

....

....

....

....

....

....

....

....

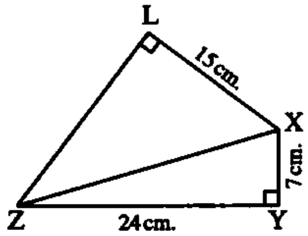
٢. **In the opposite figure :**

XYZL is a quadrilateral in which :

m (∠ XYZ) = m (∠ XLZ) = 90° ,

XY = 7 cm. , YZ = 24 cm. and XL = 15 cm.

Find the length of each of : $\overline{XZ}$ and $\overline{ZL}$



« 25 cm. , 20 cm. »

....

....

....

....

....

....

....

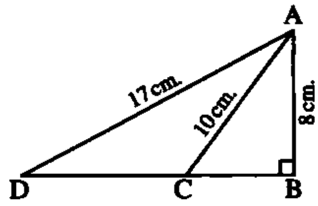
....

....

....

٣.

In the opposite figure :



Δ ABD is a right-angled triangle at B

, AB = 8 cm. , AD = 17 cm.

and C ∈ $\overline{BD}$ such that AC = 10 cm.

Find the length of each of : $\overline{CB}$, $\overline{BD}$ and $\overline{CD}$

....

....

....

....

....

....

....

....

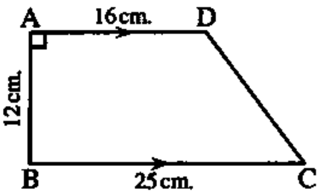
....

....

....

٤.

In the opposite figure :



ABCD is a trapezium in which $\overline{AD}$ // $\overline{BC}$

, m (∠ A) = 90° , AB = 12 cm. , BC = 25 cm. , AD = 16 cm.

Find the length of : $\overline{DC}$

....

....

....

....

....

....

....

....

....

....

....

....

Homework

٥. **In the opposite figure :**

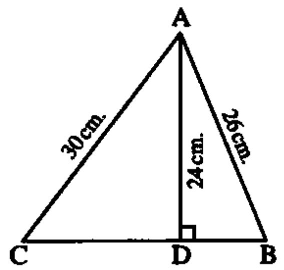
ABC is a triangle and $\overline{AD} \perp \overline{BC}$

If AD = 24 cm.

, AB = 26 cm.

and AC = 30 cm.

find BC and calculate the area of Δ ABC



« 28 cm. , 336 cm.^{2} »

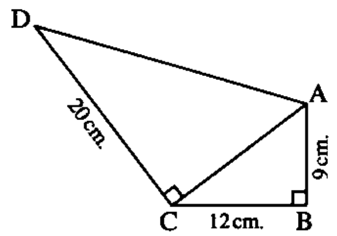
---

٦. **In the opposite figure :**

m ($\angle$ B) = m ($\angle$ ACD) = 90°

, AB = 9 cm. , BC = 12 cm.

and DC = 20 cm.



Find :

(1) The length of $\overline{AC}$

(2) The length of $\overline{AD}$

(3) The perimeter of the figure ABCD

(4) The area of the figure ABCD

٧. **In the opposite figure :**

ABC is a triangle in which

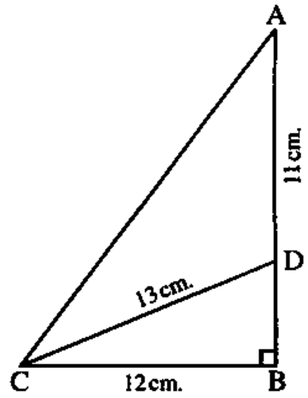
m (∠ B) = 90° ,

D ∈ $\overline{AB}$, where AD = 11 cm.

, if BC = 12 cm.

and DC = 13 cm.

Find the length of : $\overline{BD}$ and $\overline{AC}$



....

....

....

....

....

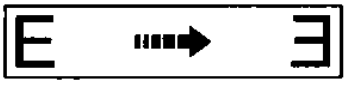
....

....

....

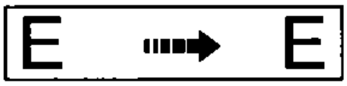
Lesson (8)
Geometric Transformations

1 Reflect its position.



Reflection

2 Move it straight a certain distance.



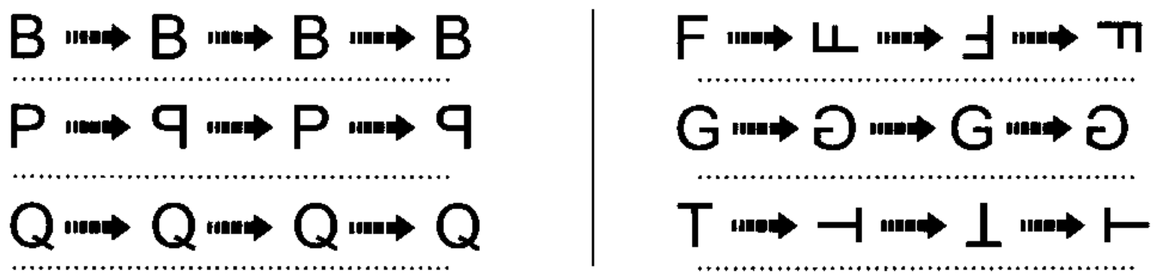
Translation

3 Rotate it with a certain angle.

Rotation anticlockwise

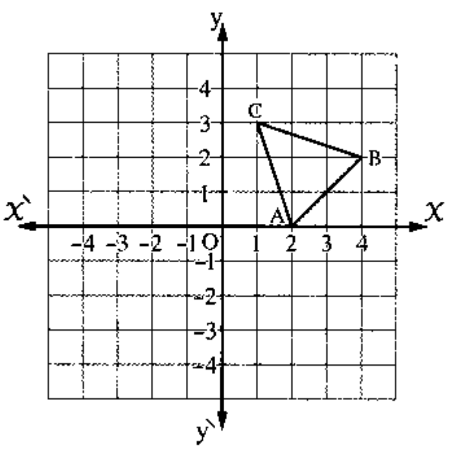
Rotation clockwise

By noticing the change that occurs to every letter compared with the figure preceding it , write down each figure the suitable word (reflection , translation , rotation) :

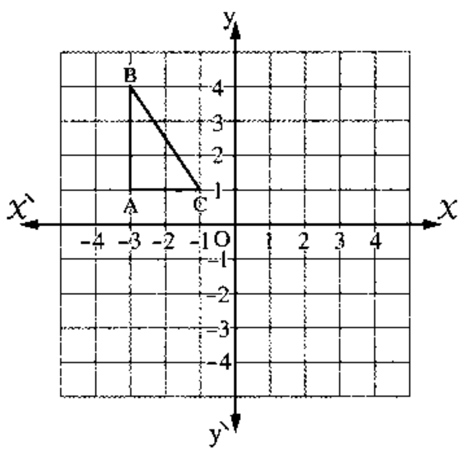


Draw the image of each of the following figures according to the illustrated geometric transformation , then describe its type :

1 $(x , y) \longrightarrow (x - 4 , y + 1)$



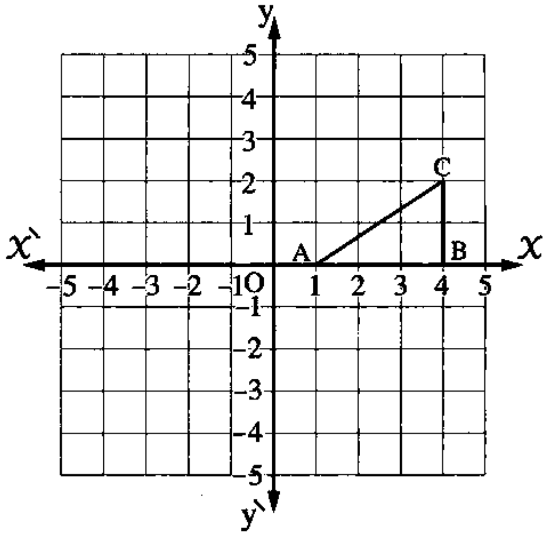
2 $(x , y) \longrightarrow (-x , -y)$



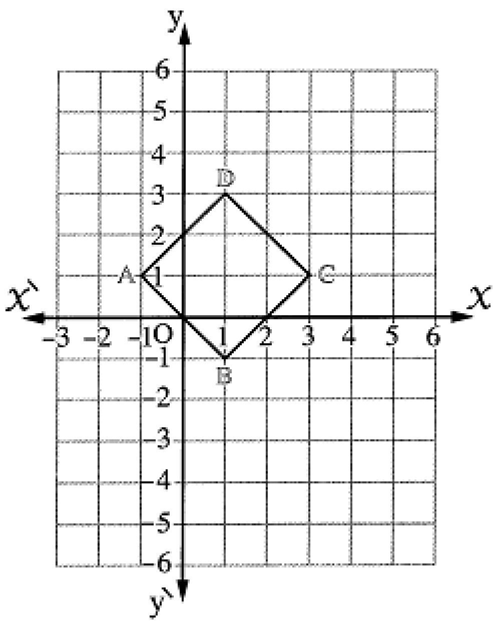
Homework

Draw the image of each figure according to the shown transformation , then describe each type :

$(x , y) \longrightarrow (-x , -y)$



$(x , y) \longrightarrow (x + 2 , y + 3)$



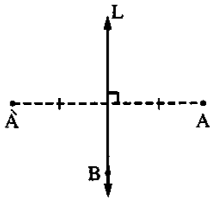
Lesson (9)
Reflection

Reflection in a straight line:

Reflection in the straight line L maps each point A to the point A\` in the same plane such that :

1. If A $\notin$ L , then the straight line L is the perpendicular bisector to the line segment $\overline{AA`}$
2. If B $\in$ L , then B is reflected onto itself

   i.e. B\` coincides B



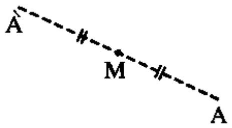
Notice that :

1. AD = AD\` , DC = D\`C\` , CB = C\`B and $\overline{AB}$ is a common side.
   i.e. Reflection in a straight line reserves the lengths of the line segments.
2. m (∠ BAD) = m (∠ BAD\`) , m (∠ ABC) = m (∠ ABC\`) , m (∠ C) = m (∠ C\`) and m (∠ D) = m (∠ D\`)
   i.e. Reflection in a straight line reserves the measures of angles.
3. From the rectangle ABCD : $\overline{AD}$ // $\overline{BC}$, from the rectangle ABC\`D\` : $\overline{AD`}$ // $\overline{BC`}$
   ∴ The images of the two parallel line segments are also two parallel line segments.
   i.e. Reflection in a straight line reserves parallelism.
4. The reading of the rectangle ABCD is in the clockwise direction while the reading of the rectangle ABC\`D\` is in anticlockwise direction.
   i.e. Reflection in a straight line doesn't reserve the orientation of the vertices of the figure.
5. If a point lies on $\overline{DC}$ and we find its image by reflection in $\overleftrightarrow{AB}$, we find its image lie on $\overline{D`C`}$
   i.e. Reflection in a straight line reserves the betweenness.

A (x , y) $\xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}}$ A\` (x , − y)	A (x , y) $\xrightarrow[\text{the y-axis}]{\text{by reflection in}}$ A\` (− x , y)

Reflection in a Point:

Reflection in a point M maps each point A in the plane to the point A' in the same plane where M is the midpoint of the line segment $\overline{AA'}$, the point M is called the centre of reflection and the image of M by reflection in M is itself.



The image of a line segment by reflection in a point is a line segment parallel to the original one and its length equals the length of the original line segment.

Notice that :

1. $A'B' = AB$, $B'C' = BC$, $C'D' = CD$ and $D'A' = DA$

   i.e. Reflection in a point reserves the lengths of the line segments.

2. $m(\angle A') = m(\angle A)$, $m(\angle B') = m(\angle B)$,
   $m(\angle B'C'D') = m(\angle BCD)$ and $m(\angle D') = m(\angle D)$

   i.e. Reflection in a point reserves the measures of angles.

3. **From the parallelogram ABCD :** $\overline{AB} \,//\, \overline{DC}$,

   From the parallelogram $A'B'C'D'$: $\overline{A'B'} \,//\, \overline{D'C'}$

   $\therefore$ The images of the two parallel line segments are also two parallel line segments.

   i.e. Reflection in a point reserves parallelism.

4. The reading of the parallelogram ABCD is in the clockwise direction and the reading of the parallelogram $A'B'C'D'$ is in the clockwise direction also.

   i.e. Reflection in a point reserves the orientation of vertices of the figure.

5. Putting a point belongs to $\overline{AB}$, we find its image by reflection in C belongs to $\overline{A'B'}$

   i.e. Reflection in a point reserves the betweenness.

The image of the point (x , y) $\xrightarrow[\text{the origin point}]{\text{by reflection in}}$ $(-x , -y)$

Complete each of the following :

١.	The number of axes of symmetry of : (a) The equilateral triangle is .... (b) The isosceles triangle is .... (c) The scalene triangle is .... (d) The parallelogram is .... (e) The rectangle is .... (f) The rhombus is .... (g) The square is .... (h) The trapezium which is not isosceles is .... (i) The isosceles trapezium .... (j) The circle ....
٢.	The reflection in a plane reserves : (a) .... (b) .... (c) .... (d) ....
٣.	If the reflection in a straight line transforms the figure to itself then this straight line is called ....
٤.	The image of the point (1 , 3) by reflection in the X-axis is ....
٥.	The image of the point (– 2 , 5) by reflection in the y-axis ....
٦.	The image of the point (2 , – 3) by reflection in the .... is (2 , 3)
٧.	The image of the point (– 1 , – 4) by reflection in the .... is (1 , – 4)
٨.	The image of the point (0 , 3) by reflection in the .... is itself.
٩.	The image of the point (– 5 , 0) by reflection in the .... is itself.
١٠.	The image of the point (2 , 1) by reflection in the X-axis followed by reflection in the y-axis is ....
١١.	The image of the point (2 , – 3) by reflection in the y-axis followed by reflection in the X-axis is ....
١٢.	The image of the point (5 , 3) by translation : $(X , y) \longrightarrow (X + 3 , y - 1)$ is ....
١٣.	(– 3 , 2) is the image of the point (3 , 2) by reflection in .... axis.
١٤.	The image of the point (4 , 6) by geometric transformation $(X , y) \longrightarrow (-X , y - 7)$ is ....

Choose the correct answer :

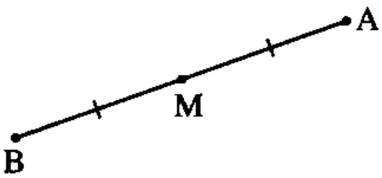
١. If $\overline{\grave{A}\grave{B}}$ is the image of $\overline{AB}$ by reflection in M , then $\grave{A}\grave{B}$ .... AB

(a) > (b) < (c) = (d) ≠

٢. **In the opposite figure :**

The image of $\overline{AB}$

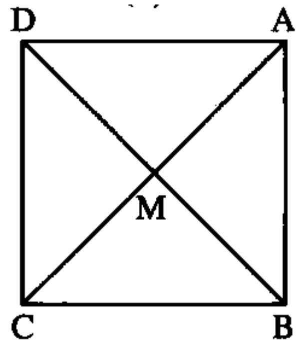
by reflection in the point M is ....



(a) $\overline{AM}$ (b) $\overline{AB}$ (c) $\overline{BA}$ (d) $\overline{BM}$

٣. **In the opposite figure :**

ABCD is a square whose diagonals intersect at M The image of Δ ABM by reflection in M is Δ ....



(a) ADM (b) BCM
(c) DCM (d) CDM

٤. If $\grave{A}$ is the image of A by reflection in M and if MA = 5 cm. , then $A\grave{A}$ = ...

(a) 5 cm. (b) 7 cm. (c) 10 cm. (d) 15 cm.

٥. The image of the point (– 3 , 2) by reflection in the origin point is ....

(a) (3 , 2) (b) (– 3 , – 2) (c) (3 , – 2) (d) (– 3 , 2)

٦. The point (5 , – 2) is the image of the point .... by reflection in the origin point.

(a) (5 , – 2) (b) (– 5 , – 2) (c) (– 5 , 2) (d) (5 , 2)

٧. The point whose image by reflection in the origin point is itself is ....

(a) (0 , 1) (b) (1 , 0) (c) (0 , 0) (d) (– 1 , 0)

٨. The image of the point (3 , – 2) by reflection in the origin point followed by reflection in x-axis is ....

(a) (3 , – 2) (b) (– 3 , – 2) (c) (– 3 , 2) (d) (3 , 2)

٩. The image of the point (2 , – 5) by reflection in x-axis is ....

(a) (2 , – 5) (b) (2 , 5) (c) (– 2 , – 5) (d) (5 , 2)

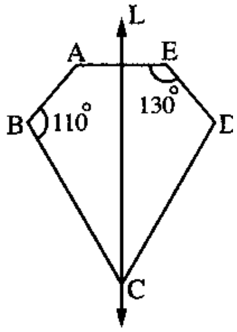
١٠. The image of the point (3 , – 5) by reflection in y-axis is ....

(a) (3 , 5) (b) (– 3 , – 5) (c) (– 3 , 5) (d) (– 5 , 3)

Essay problems:

١. **In the opposite figure :**

If the straight line L
is the axis of symmetry
of the figure ABCDE ,
calculate : m (∠ BCD)



« 60° »

....

....

....

....

....

....

٢. Draw $\overline{AB}$ where A (4 , 3) and B (1 , – 2) , then draw its image by reflection in :

(1) The X-axis   (2) The y-axis

٣. Find the image of Δ ABC where A (– 6 , – 1) , B (– 2 , – 1) and C (– 5 , – 6) by reflection in the X-axis

٤. In Xy - coordinate plane , draw Δ ABC, where : A (– 2 , 4) , B (5 , 0) and C (3 , – 3) , then find the reflected image of Δ ABC in the origin point.

Homework

٥. 📖 Draw the image of the square ABCD where A (2 , 3) and B (2 , – 1) by reflection in the y-axis. What do you notice ?

٦. 📖 ABCD is a rectangle where A (2 , 5) , B (6 , 5) , C (6 , 8) and D (2 , 8) , then find the image of the rectangle ABCD by reflection in the origin point.

Lesson (10)
Translation

Translation is a geometrical transformation which maps each point A in the plane to another point A' in the same plane with a constant distance in a certain direction.

Notice that :

1 $A'B' = AB$, $AB = DC$

i.e. Translation reserves the lengths of the line segments.

2 m (∠ A') = m (∠ BAD) , m (∠ B') = m (∠ CBA)

i.e. Translation reserves the measures of angles.

3 From the square ABCD : $\overline{AB}$ // $\overline{DC}$, from the square $A'B'BA$: $\overline{A'B'}$ // $\overline{AB}$

∴ The images of the two parallel line segments are also two parallel line segment.

i.e. Translation reserves the parallelism.

4 The reading of the square ABCD is in the clockwise direction and the reading of the square $A'B'BA$ is in the clockwise direction also.

i.e. Translation reserves the orientation of vertices of the figure.

5 If you take a point lies on $\overline{AB}$ and find its image by the previous translation , you will find its image lies on $\overline{A'B'}$

i.e. Translation reserves the betweenness.

Translation in the orthogonal Cartesian coordinates plane transforms each point by a displacement a in the direction of the x-axis followed by a displacement b in the direction of the y-axis

i.e. The image of the point A (x , y) ⟶ the point A' (x + a , y + b)

Complete each of the following :

١.	The image of the point (2 , 5) by translation (x , y) ⟶ (x + 2 , y + 1) is ………
٢.	The image of the point (3 , 2) by translation (x , y) ⟶ (x + 3 , y − 2) is ………

٣.	The image of the point (– 5 , 4) by translation (x , y) ⟶ (x + 4 , y – 5) is ………
٤.	The image of the point (– 2 , – 5) by translation (x , y) ⟶ (x – 2 , y) is ………
٥.	The image of the point (3 , – 2) by translation (x , y) ⟶ (x , y + 3) is ………

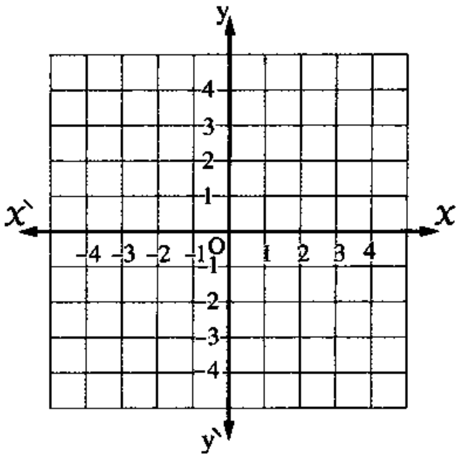
٦. **On a square lattice , draw Δ ABC where A (– 3 , 2) , B (– 1 , 1) , C (– 2 , 0) , then find its image by translation :**

(x , y) ⟶ (x + 2 , y + 1)

À = (……… , ………)

B̀ = (……… , ………)

C̀ = (……… , ………)



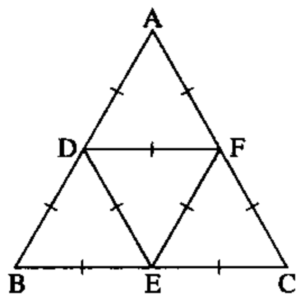
٧. **In the opposite figure :**

The triangles ADF , BDE , DEF and EFC are congruent.

Complete :

(1) The image of Δ ADF by a translation of magnitude of AD in the direction of $\overrightarrow{AD}$ is ………

(2) Δ FEC is the image of Δ DBE by a translation of magnitude ……… in the direction of ………



٨. **In the opposite figure :**

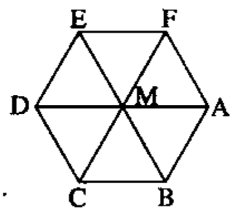
ABCDEF is a regular hexagon. **Complete the following :**

(1) The image of the point D by translation DM in the direction of $\overrightarrow{DM}$ is ………

(2) The image of $\overline{AF}$ by translation ED in the direction of $\overrightarrow{ED}$ is ………

(3) The image of Δ MCD by translation EF in the direction of $\overrightarrow{EF}$ is ………

(4) The translation which makes Δ DME the image of Δ MAF is ………



Choose the correct answer :

١. The image of the point (– 1 , 2) by translation of magnitude of 3 units in the positive direction of the x-axis is ………

(a) (– 1 , 5) (b) (2 , 2) (c) (– 2 , 2) (d) (– 1 , 3)

٢. The image of the point (– 3 , 4) by translation of magnitude of 4 units in the negative direction of the y-axis is ....

(a) (– 3 , 0) (b) (– 7 , 4) (c) (– 3 , 8) (d) (– 1 , 4)

٣. If À (3 , – 3) is the image of A by translation $(x , y) \longrightarrow (x - 1 , y - 4)$, then the point A is ....

(a) (2 , – 7) (b) (4 , 1) (c) (– 4 , – 1) (d) (2 , 1)

٤. The image of the point (– 1 , 4) by the translation (3 , – 2) followed by reflection in the x-axis is ....

(a) (2 , 2) (b) (– 2 , 2) (c) (– 2 , – 2) (d) (2 , – 2)

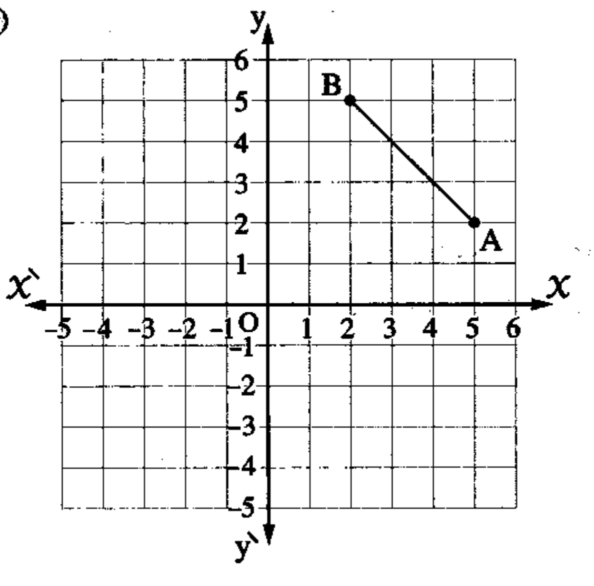
٥. If the point (a , – 1) is the image of (2 , 4) by the translation $(x , y) \longrightarrow (x + 1 , y - b)$, then (a , b) is ....

(a) (3 , 3) (b) (1 , 3) (c) (3 , 5) (d) (1 , – 5)

Essay problems:

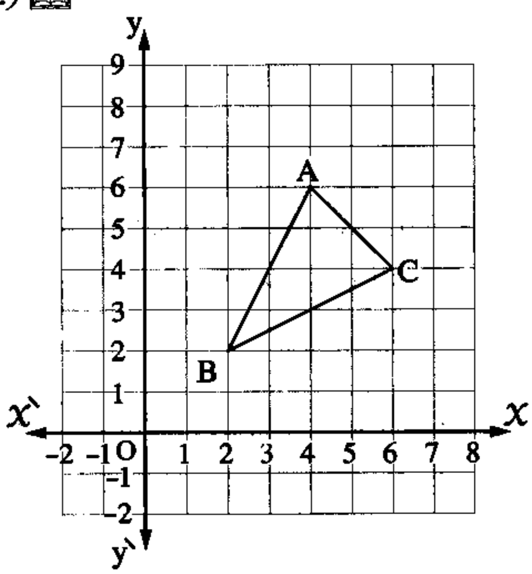
١. **Find the image of each of the following figures by the translation shown under each figure :**

(1)



$(x , y) \longrightarrow (x - 3 , y - 4)$

(2)



$(x , y) \longrightarrow (x + 2 , y + 3)$

....

....

....

٢. Using the geometric instruments , draw the square ABCD whose side length is 4 cm. , then draw its image by translation of magnitude of 4 cm. in the direction of $\overrightarrow{AB}$

٣. 📖 Using the lattice , find the image of each of the following points by the translation of LM in the direction of $\overrightarrow{LM}$ where : L (1 , 3) and M (4 , 5)

(1) B (– 2 , 3) (2) C (5 , 4) (3) D (3 , 0)

٤. If the image of the point A (1 , 1) by translation in the plane is À (2 , 2) , find the images of the points O (0 , 0) , B (– 1 , 3) and C (– 3 , 5) by the same translation.

Homework

٥. Draw a line segment $\overline{AB}$ where AB = 5 cm. , then draw the image of $\overline{AB}$ by a translation of magnitude of 8 cm. in the direction of $\overrightarrow{AB}$

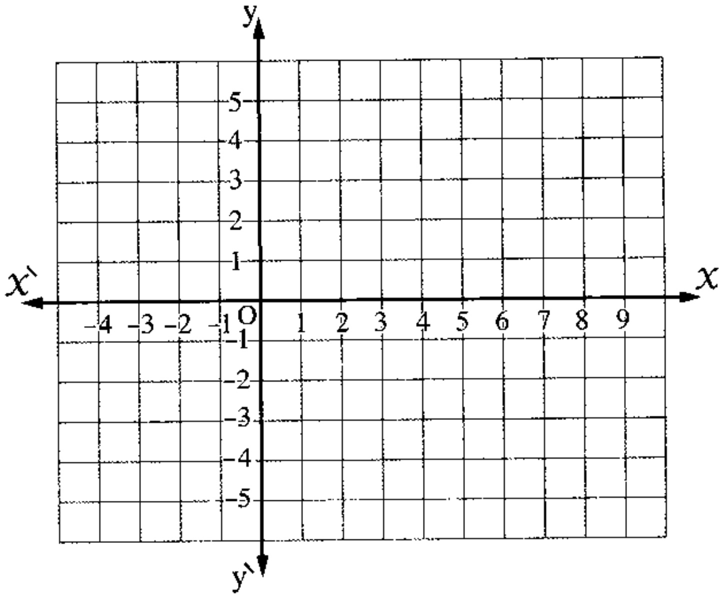
٦. Use the translation : $(x , y) \longrightarrow (x + 2 , y + 3)$ to locate the point whose image is (2 , 3)

....

....

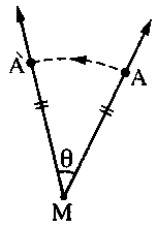
....

٧. Draw the square ABCD where A (4 , – 2) , B (4 , – 5) , C (1 , – 5) and D (1 , – 2) , then find its image by translation CA in the direction of $\overrightarrow{CA}$



Lesson (11)
Rotation

If M is a fixed point in the plane, then the rotation around M with an angle of measure θ is a geometric transformation transforming each point A in the plane to another point À in the same plane such that m (∠ AMÀ) = θ , MA = MÀ



It is denoted by R (M , θ) where :

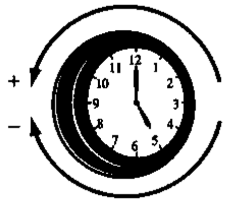
- M is the centre of rotation.
- θ is the measure of the angle of rotation.

According to this concept , the rotation is determined completely if we know the following elements :

1. The centre of rotation.
2. The measure of the angle of rotation (θ)
3. The direction of rotation.

Remark

The measure of rotation angle is positive if the rotation is anticlockwise and it is negative if the rotation is clockwise.



Rotation in the plane reserves the lengths of the line segments.
Rotation in the plane reserves the measures of angles.
Rotation in the plane reserves the orientation of vertices of the figure.
Rotation in the plane reserves parallelism.
Rotation in the plane reserves the betweenness.
Rotation in the plane reserves the collinearity.

The image of the point $(x , y) \xrightarrow[R(O,\ 90°)]{\text{by rotation}}$ the point $(-y , x)$

The image of the point $(x , y) \xrightarrow[R(O,\ -90°)]{\text{by rotation}}$ the point $(y , -x)$

Rotation about the origin point with an angle of measure 270° is equivalent to rotation about the origin point with an angle of measure (– 90°)

The image of the point $(x , y) \xrightarrow[R(O,\ 180°)]{\text{by rotation}}$ the point $(-x , -y)$

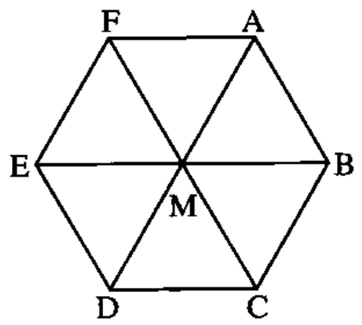
Remarks

1. The image of the point A (x , y) by rotation R (O , 180°) is the same image of the point A by rotation R (O , – 180°)
2. The image of the point A (x , y) about the origin point with an angle of measure ± 360° is the same point A (x , y)
3. Rotation with an angle of measure 90° is called a $\frac{1}{4}$ turn.
4. Rotation with an angle of measure 180° is called a $\frac{1}{2}$ turn.
5. Rotation with an angle of measure 360° is called the identity rotation because it returns the figure to its original position.

Complete each of the following :

١. **In the opposite figure :**

ABCDEF is a regular hexagon. Complete the following :



1. The image of the point A by rotation around M with an angle of measure 180° is ....
2. The image of $\overline{AB}$ by rotation around M with an angle of measure (– 60°) is ....
3. The image of Δ CMD by rotation around M with an angle of measure 120° is ....

٢. The image of the point (2 , – 3) by rotation about the origin point with an angle of measure 90° is .... and with an angle of measure 180° is ....

٣. The image of the point (– 1 , 0) by rotation about the origin point with an angle of measure 90° is .... and with an angle of measure 360° is ....

٤. The point (3 , – 2) is the image of the point (2 , 3) by rotation about the origin point with an angle of measure ....

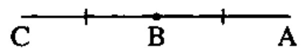
٥. The image of the point .... by rotation about the origin point with an angle of measure 90° is (– 1 , 4)

٦. The image of the point .... by rotation about the origin point with an angle of measure (– 180°) is (5 , – 2)

٧.	The image of the point (– 3 , 7) by rotation 90° about the origin point followed by reflection in y-axis is ……….
٨.	The image of the point (– 2 , 0) by translation $(x , y) \longrightarrow (x + 3 , y - 1)$ followed by rotation about the origin point with an angle of measure 90° is ……….
٩.	The rotation with an angle of measure 90° about the origin point maps the point $(x , -y)$ onto the point ……….
١٠.	The image of (a , b) is the same point by rotation about the origin point with an angle of measure ……….
١١.	If the image of the point (x , y) by rotation about the origin point with an angle of measure 90° is (a , b) , then a + y = ……….

Choose the correct answer :

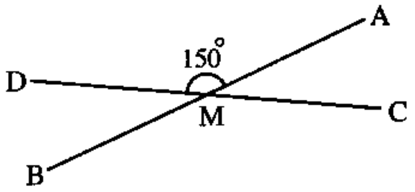
١. **In the opposite figure :**



If B is the midpoint of $\overline{AC}$, then the image of $\overline{AC}$ by rotation about B with an angle of 180° is ……….

(a) $\overline{AC}$ (b) $\overline{AB}$ (c) $\overline{CA}$ (d) $\overline{CB}$

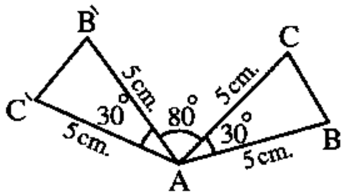
٢. **In the opposite figure :**



$\overline{CD}$ is the image of $\overline{AB}$ under a rotation about M and the measure of its angle is ……….

(a) 75° (b) 30° (c) – 30° (d) – 150°

In the opposite figure :



Δ AB̀C̀ is the image of Δ ABC

٣. by a rotation about A

with an angle of measure ....

(a) – 110°   (b) 80°   (c) 110°   (d) 140°

Essay problems:

Draw on graph paper Δ ABC where A (3 , – 1) , B (5 , 2) and C (– 2 , 4) , then draw its image by rotation about the origin point with an angle of measure 180°

١.

٢٠

In an orthogonal Cartesian coordinated system , determine the two points A (3 , 0) and B (0 , 2) , then draw the image of Δ AOB by rotation about «O» with an angle of measure 90° where O is the origin point.

Summary

The image of the point (x , y)

by reflection in the x-axis	The point (x , – y)
by reflection in the y-axis	The point (– x , y)
by reflection in the origin point	The point (– x , – y)
by translation (x, y) ⟹ (x + k , y + ℓ)	The point (x + k , y + ℓ)
by rotation R (O , 90°) ($\frac{1}{4}$ turn)	The point (– y , x)
by rotation about O with an angle of measure (– 90°) or (270°)	The point (y , – x)
by rotation R (O , ± 180°) ($\frac{1}{2}$ turn)	The point (– x , – y)
by rotation R (O , ± 360°) (identity rotation)	The point (x , y)

Best Wishes