Choose the correct answer:

1 $(x+1)^2 = \dots\dots\dots$

(a) x^2+1 (b) x^2-1 (c) x^2-x+1 x^2+2x+1

2 If A , B are two mutually exclusive events , P (B) = 0.5 and P (A ∪ B) = 0.7 , then P (A) = $\dots\dots\dots$

(a) 0.02 0.2 (c) 0.5 (d) 0.13

3 If the two equations $x+2y=1$, $2x+ky=2$ has only one solution , then $k \neq \dots\dots\dots$

(a) 1 (b) 2 4 (d) – 4

4 The domain of the function f : $f(x) = \frac{x-3}{4}$ is $\dots\dots\dots$

$\mathbb{R}$ (b) $\mathbb{R}-\{-4\}$ (c) $\mathbb{R}-\{-4, 3\}$ (d) $\varnothing$

5 If $A \subset S$ of a random experiment , $P(A) = P(A')$, then $P(A) = \dots\dots\dots$

(a) 1 $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

6 The set of zeroes of the function f : where $f(x) = -3x$ is $\dots\dots\dots$

$\{0\}$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\mathbb{R}-\{3\}$

7 In the experiment of rolling a regular die once , the probability of appearance of an even number on the upper face = $\dots\dots\dots$

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ $\frac{1}{2}$ (d) $\frac{5}{6}$

8 If $x^2 - y^2 = 2(x+y)$ where $(x+y) \neq$ zero , then $(x-y) = \dots\dots\dots$

2 (b) 4 (c) 6 (d) 8

9 If $P(A) = 4P(A')$, then $P(A) = \dots\dots\dots$

0.8 (b) 0.6 (c) 0.4 (d) 0.2

10 The domain of the function f where $f(x) = \frac{x+2}{5x}$ is $\dots\dots\dots$

(a) $\mathbb{R}-\{5\}$ (b) $\mathbb{R}-\{-5\}$ (c) $\mathbb{R}$ $\mathbb{R}-\{\text{zero}\}$

11 Twice the number X subtracted by 3 is ....

(a) $X - 3$ (b) $2X + 3$ (c) $2X - 3$ $3 - 2X$

12 The point of intersection of the two straight lines $X = 2$ and $X + y = 6$ is ....

(a) (2 , 6) (2 , 4) (c) (4 , 2) (d) (6 , 2)

13 If $X = 2$ and $y = 3$, then $(y - 2X)^{10} =$ ....

(a) 10 (b) – 1 (c) – 10 1

14 If X is a negative real number , then the greatest number of the following numbers is ....

(a) $3 + X$ (b) $3X$ $3 - X$ (d) $\frac{3}{X}$

15 The additive inverse of the fraction $\frac{3}{X^2 + 1}$ is ....

$\frac{-3}{X^2 + 1}$ (b) $\frac{X^2 + 1}{3}$ (c) $\frac{X^2 + 1}{-3}$ (d) $\frac{3}{X^2 - 1}$

16 If A and B are two mutually exclusive events of random experiment then :
$P(A \cap B) =$ ....

(a) $P(A \cup B)$ (b) $P(A) + P(B)$ (c) $\varnothing$ zero

17 If $f(X) = 6X^2 + 3X(1 - 2X)$ is a polynomial function , then its degree is ....

first. (b) second. (c) third. (d) fourth.

18 The set of zeroes of the function $f : f(X) = \frac{X^2 - 9}{X - 3}$ is ....

(a) {3} {– 3} (c) {3 , – 3} (d) $\varnothing$

19 If the curve of the quadratic function does not intersect the X-axis at any point , then the number of solutions of the equation $f(X) = 0$ in $\mathbb{R}$ is ....

zero (b) one solution. (c) two solutions. (d) an infinite number.

20 If $a < \sqrt{3} < b$, then (a , b) is ....

(a) (0 , 1) (b) (2.5 , 3.5) (1 , 2) (d) (2 , 3)

21 If $(7^{a-2}, 3) = (1, b+5)$, then $a + b =$ ……………

(a) -1 zero (c) 1 (d) 2

22 The domain of the multiplicative inverse of the algebraic fraction $\frac{x+2}{x+5}$ is ……………

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{-5\}$ (c) $\mathbb{R} - \{-2\}$ $\mathbb{R} - \{-2, -5\}$

23 If $2^5 \times 3^5 = m \times 6^4$, then $m =$ ……………

(a) 1 (b) 2 (c) 3 6

24 The point of intersection of the two straight lines $x + 2 = 0$ and $y - 3 = 0$ is ……………

(a) $(-2, -3)$ $(-2, 3)$ (c) $(2, -3)$ (d) $(2, 3)$

25 The set of zeroes of the function $f : f(x) = x^2 + 1$ is ……………

(a) $\{1\}$ (b) $\{-1\}$ (c) $\{-1, 1\}$ $\varnothing$

26 If the curve of the quadratic function f passes through the points $(2, 0)$, $(-3, 0)$ and $(0, -6)$, then the solution set of the equation $f(x) = 0$ in $\mathbb{R}$ is ……………

(a) $\{-2, 3\}$ (b) $\{3, 2\}$ $\{2, -3\}$ (d) $\{-3, -6\}$

27 If there is only one solution for the equation :

$x + 2y = 1$ and $2x + ky = 2$ in $\mathbb{R} \times \mathbb{R}$, then k cannot equal ……………

(a) 2 4 (c) -2 (d) -4

28 If $n(x) = \frac{x+1}{x-2}$ is an algebraic fraction, then the domain in which the fraction has multiplicative inverse is ……………

(a) $\mathbb{R} - \{2\}$ $\mathbb{R} - \{-1, 2\}$ (c) $\mathbb{R} - \{-1\}$ (d) $\{-1, 2\}$

29 If the curve of the function $f : f(x) = x^2 - x + c$ passing through the point $(2, 1)$, then $c =$ ……………

(a) 2 (b) 1 (c) -2 -1

30 The set of zeroes of the function $f : f(x) = \frac{x^2 - x - 2}{x^2 + 4}$ is ……………

(a) $\{2, -2\}$ (b) $\{-2, -1\}$ $\{2, -1\}$ (d) $\{1, -1\}$

31	If A and B are two mutually exclusive events of a random experiment , if $P(A) = \frac{1}{3}$, $P(A \cup B) = \frac{7}{12}$, then P (B) = ... $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1
32	If $n(X) = \frac{X+2}{X-5}$, then the domain of n^{-1} is .... (a) $\{2 , -5\}$ (b) $\{-2 , 5\}$ $\mathbb{R} - \{-2 , 5\}$ (d) $\mathbb{R} - \{2 , -5\}$
33	If the solution set of the equation $X^2 - aX + 4 = 0$ is $\{-2\}$, then a = ... (a) – 2 – 4 (c) 2 (d) 4
34	The simplest form of the function n : $n(X) = \frac{3-X}{X-3}$ such that $X \in \mathbb{R} - \{3\}$ is .... (a) 1 – 1 (c) 3 (d) – 3
35	The function f where $f(X) = \frac{X-3}{X-4}$ has additive inverse in the domain .... (a) $\mathbb{R} - \{3\}$ $\mathbb{R} - \{4\}$ (c) $\mathbb{R} - \{-4\}$ (d) $\mathbb{R} - \{-3\}$
36	If $A \subset S$ of a random experiment , $P(A) = P(\grave{A})$, then P (A) = ... (a) zero (b) 1 $\frac{1}{2}$ (d) $\frac{1}{4}$
37	The set of zeroes of f where $f(X) = (X-1)^2 (X+2)$ is .... $\{1 , -2\}$ (b) $\{-1 , 2\}$ (c) $\{-1 , -2\}$ (d) $\{1 , 2\}$
38	If the curve of the function f where $f(X) = X^2 - a$ passes through the point (1 , 0) , then a = ... (a) – 2 (b) – 1 (c) zero 1
39	If A and B are two mutually exclusive events , then P (A – B) = ... (a) zero P (A) (c) P (B) (d) $P(A \cup B)$
40	Number of solutions of the two equations : $X + y = 2$, $y - 3 = 0$ together is .... (a) 3 (b) 2 1 (d) zero

41 If $(5, x-4) = (y, 3)$, then $x + y =$ ....

(a) 25 12 (c) 8 (d) 6

42 The set of zero is of f where $f(x) = x(x^2 - 2x + 1)$ is ....

$\{0, 1\}$ (b) $\{0, -1\}$ (c) $\{-1, 1\}$ (d) $\{0, 1, -1\}$

43 If the age of a man now is x year, then his age after 5 years from now is .... years.

(a) $x - 5$ (b) $5 - x$ (c) $5x$ $x + 5$

44 If A is an event of random experiment, then P (Á) = ...

(a) 1 (b) – 1 1 – P (A) (d) P (A) – 1

45 If $2x^2 = 5$, then $6x^2 =$ ....

(a) 5 (b) 10 15 (d) 20

46 The sum of two consecutive integers is 17, then the smaller number of them is ....

8 (b) 9 (c) 17 (d) 72

47 The probability of the impossible event equals ....

(a) ∅ (b) 1 zero (d) – 1

48 The sum of two consecutive integers is 17, then the smaller number of them is ....

8 (b) 9 (c) 17 (d) 72

49 If $2x^2 = 5$, then $6x^2 =$ ....

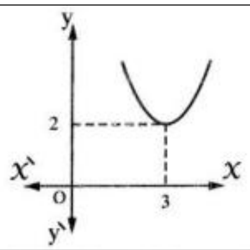
(a) 5 (b) 10 15 (d) 20

50 **In the opposite figure :**

The solution set of $f : f(x) = 0$ is ....

∅ (b) $\{3\}$

(c) $\{2, 3\}$ (d) $\{2\}$



51 If the two equations : $x + 3y = 6$, $2x + ky = 12$ have an infinit number of solutions , then k = ...

(a) 1 [redacted] 6 (c) 3 (d) 2

52 $\sqrt{16+9} = 4 +$ ....

(a) 3 (b) 5 [redacted] 1 (d) 7

53 If A and B are two mutually exclusive events of a random experiment , then P (A ∩ B) = ...

(a) ∅ (b) 1 [redacted] zero (d) $\frac{1}{2}$

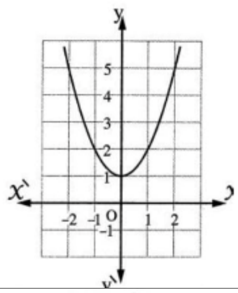
54 $|-5| =$ ....

(a) – 5 (b) $-\frac{1}{5}$ [redacted] 5 (d) $\frac{1}{2}$

55 If A ⊂ S , $P(A) = \frac{1}{3}$, then P (À) = ...

(a) $\frac{1}{3}$ [redacted] $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

56 The opposite figure represents the curve of a quadratic function f , then the solution set of the equation $f(x) = 0$ is ....



[redacted] ∅ (b) {1}

(c) {0} (d) {(0 , 1)}

57 If the fraction $\frac{x - a}{x + 3}$ is the multiplicative inverse of $\frac{x+3}{x+5}$, then a = ...

[redacted] – 5 (b) – 3 (c) 3 (d) 5

58 The set of zeroes of the function $f : f(x) = 4$ is ....

(a) {– 4} (b) {zero} [redacted] ∅ (d) {2}

59 The solution set of the equation : $x^2 - 4x + 4 = 0$ in ℝ is ....

(a) {– 2} [redacted] {2} (c) {4 , 1} (d) ∅

60 If the probability that a student succeeded is 95 % , then the probability that he does not succeed is ....

(a) 20 % [redacted] 5 % (c) 10 % (d) zero

61	If $x + y = 5$, then $3x + 3y =$ …………… (a) 5 (b) 3 (c) 8 15
62	If $3^{x} = 1$, then $x =$ …………… 0 (b) $\frac{1}{3}$ (c) 1 (d) 3
63	If $\grave{A}$ is the complement event of the event A in a sample space of a random experiment , then $P(A) + P(\grave{A}) =$ …………… (a) 2 1 (c) $\frac{1}{2}$ (d) 3
64	$x^2 + kx + 9$ is a perfect square if $k =$ …………… (a) 3 (b) -3 (c) ± 3 ± 6
65	If $P(A) = 4P(\grave{A})$, then $P(A) =$ …………… 0.8 (b) 0.6 (c) 0.4 (d) 0.2
66	If $y^{-3} = 8$, then $y =$ …………… (a) $\frac{1}{512}$ (b) 1 (c) 2 $\frac{1}{2}$
67	The two straight lines : $x + 5y = 1$, $x + 5y - 8 = 0$ are …………… parallel. (b) coincide. (c) perpendicular. (d) intersect and non perpendicular.
68	If A , B are two events from the sample of a random experiment , $P(A) = 0.7$ and $P(A - B) = 0.5$, then $P(A \cap B) =$ …………… (a) 0.6 (b) 0.4 (c) 0.3 0.2
69	If $x^2 - y^2 = 15$ and $x - y = 3$, then $x + y =$ …………… (a) -5 (b) -3 (c) 3 5
70	[2 , 5] is the solution set of the inequality …………… $1 \le x - 1 \le 4$ (b) $1 < x - 1 < 4$ (c) $1 \le x - 1 < 4$ (d) $1 < x - 1 \le 4$

71	If : $\frac{1}{x} + \frac{1}{y} + \frac{1}{xy} = \frac{\cdots\cdots}{yx}$ (a) 2 (b) 3 $x + y + 1$ (d) $x + y$
72	If $\frac{1}{3}x - \frac{5}{12} = \frac{1}{4}$, then $x = \cdots\cdots$ (a) $\frac{2}{3}$ 2 (c) $\frac{7}{12}$ (d) $\frac{3}{4}$
73	If $\frac{x}{y} = \frac{3}{4}$, then $\frac{4x}{3y} = \cdots\cdots$ (a) $\frac{4}{3}$ 1 (c) $\frac{9}{16}$ (d) $\frac{16}{9}$
74	If the ratio between the perimeters of two squares is 1 : 2 , then the ratio between their areas = ... : .... (a) 1 : 2 (b) 2 : 1 1 : 4 (d) 4 : 1
75	If $(x - 5)^{zero} = 1$, for every $x \in$ .... (a) $\mathbb{R}$ $\mathbb{R} - \{5\}$ (c) $\mathbb{R} - \{-5\}$ (d) $\mathbb{R} - \{1\}$
76	If A B = 3 , $AB^2 = 12$, then B = ... (a) 2 (b) 3 4 (d) 5
77	If the function f is a function from set X to set Y , then the domain of the function is .... X (b) Y (c) X × Y (d) Y × X
78	$(-1)^{99} + (-1)^{100} = \cdots\cdots$ (a) – 2 zero. (c) 1 (d) 2
79	If a die is rolled once , then the probability of getting an odd number and even number together = ... (a) $\frac{1}{2}$ zero. (c) $\frac{3}{4}$ (d) 1
80	If x is a negative number , then the greatest number from the following numbers is .... $5 - x$ (b) $5 + x$ (c) $\frac{5}{x}$ (d) $5x$

1 $x + y = 4$, $x - y = 2$

Solution

$x + y = 4$

$x - y = 2$

$2x = 6$ $\quad \therefore x = 3$

$\therefore 3 + y = 4$ $\quad \therefore y = 1$

$\therefore$ The S.S. $= \{(3, 1)\}$

2 $x = y$, $x + 3y = 8$

Solution

$\because x = y$ (1) , $x + 3y = 8$ (2)

Substituting from (1) in (2) : $\therefore y + 3y = 8$

$\therefore 4y = 8$ $\quad \therefore y = 2$

Substituting in (1) : $\therefore x = 2$

$\therefore$ The S.S. $= \{(2, 2)\}$

3 $x + 3y = 2$, $3x + 4y = 6$

Solution

$x + 3y = 2$ multiplying by -3

$3x + 4y = 6$

$-3x - 9y = -6$ (1)

, $3x + 4y = 6$ (2)

$\therefore -5y = \text{zero}$ $\quad \therefore y = \text{zero}$

Substituting in (1) : $\therefore x = 2$

$\therefore$ The S.S. $= \{(2, 0)\}$

4 $3x + 4y = 24$, $x - 2y = -2$

Solution

$3x + 4y = 24$

$x - 2y = -2$ multiplying by 2

$3x + 4y = 24$

$\therefore 2x - 4y = -4$

$\therefore 5x = 20$ $\quad \therefore x = 4$

$12 + 4y = 24$ $\quad \therefore y = 3$

The S.S. $= \{(4, 3)\}$

5 $x - 3y = 6$, $2x + y = 5$

Solution

$\because x - 3y = 6$ $\quad \therefore x = 6 + 3y$

$\because 2x + y = 5$

$\therefore 2(6 + 3y) + y = 5$ $\quad \therefore 12 + 6y + y = 5$

$\therefore 7y = -7$ $\quad \therefore y = -1$

Substituting in (1) : $\therefore x = 6 - 3 = 3$

$\therefore$ The S.S. $= \{(3, -1)\}$

6 If $f(x) = ax^2 + b$, $f(1) = 5$, $f(2) = 11$ then find the value of a and b

Solution

$\because f(x) = ax^2 + b$, $f(1) = 5$

$\therefore a + b = 5$ (1) , $\because f(2) = 11$

$\therefore 4a + b = 11$ (2)

Subtracting (1) from (2) : $\therefore 3a = 6$ $\quad \therefore a = 2$

Substituting in (1) : $\therefore b = 3$

7 If $f(x) = \dfrac{x^2 - 9}{x + b}$, $f(4) = 1$ **Find :** b

Solution

$\because f(4) = 1$ $\quad \therefore \dfrac{16 - 9}{4 + b} = 1$

$\therefore 7 = 4 + b$ $\quad \therefore b = 3$

8

Solution

If (a , 2 b) is a solution for the two equations :

$\because$ (a , 2 b) is a solution for the equation : $3x - y = 5$

$\therefore 3a - 2b = 5$ (1)

, $\because$ (a , 2 b) is a solution for the equation : $x + y = -1$

$\therefore a + 2b = -1$ (2)

Adding (1) and (2) : $\therefore 4a = 4$ $\therefore a = 1$

Substituting in (1) : $\therefore b = -1$

9

Find the value of a and b

$\{(3, -1)\}$ is the solution set of the two

$a x + b y - 5 = 0$ $3 a x + b y = 17$

Solution

$\because$ (3 , – 1) is a solution for the equation

$a x + b y - 5 = 0$ $\therefore 3a - b = 5$ (1)

$\because$ (3 , – 1) is a solution for the equation

$3 a x + b y = 17$ $\therefore 9a - b = 17$

$\therefore -9a + b = -17$ (2)

Adding (1) and (2) : $\therefore -6a = -12$ $\therefore a = 2$

Substituting in (1) : $\therefore b = 1$

10

$x + y = 2$, $\frac{1}{x} + \frac{1}{y} = 2$, where $x \neq 0$, $y \neq 0$

Solution

$\because x + y = 2$ $\therefore x = 2 - y$ (1)

$\because \frac{1}{x} + \frac{1}{y} = 2$ $\therefore y + x = 2xy$

$\therefore y + x - 2xy = 0$ (2)

Substituting from (1) in (2) :

$\therefore y + 2 - y - 2y(2 - y) = 0$

$\therefore 2y^2 - 4y + 2 = 0$

$\therefore y^2 - 2y + 1 = 0$ $\therefore (y - 1)^2 = 0$ $\therefore y = 1$

From (1) : $\therefore x = 1$

$\therefore$ The S.S. $= \{(1, 1)\}$

11

$x + y = 0$, $y^2 = x$

Solution

Substituting from equ. (2) in equ. (1) :

$\therefore y^2 + y = 0$ $\therefore y(y + 1) = 0$

$\therefore y = 0$ or $y = -1$

Substituting in equ. (1) : $\therefore x = 0$ or $x = 1$

$\therefore$ The S.S. $= \{(0, 0), (1, -1)\}$

12

$x + y = 5$, $\frac{xy}{6} = 1$

Solution

$\because x + y = 5$ $\therefore y = 5 - x$ (1)

$\because \frac{xy}{6} = 1$ $\therefore xy = 6$ (2)

Substituting from (1) in (2) :

$\therefore x(5 - x) = 6$ $\therefore 5x - x^2 - 6 = 0$

$\therefore x^2 - 5x + 6 = 0$ $\therefore (x - 2)(x - 3) = 0$

$\therefore x = 2$ or $x = 3$

And from (1) : $\therefore y = 3$ or $y = 2$

$\therefore$ The S.S. $= \{(2, 3), (3, 2)\}$

13

$x + y = 7$, $xy = 12$

Solution

$\because x + y = 7$ $\therefore y = 7 - x$ (1)

Substituting in the second equation :

$\therefore x(7 - x) = 12$ $\therefore 7x - x^2 = 12$

$\therefore x^2 - 7x + 12 = 0$ $\therefore (x - 3)(x - 4) = 0$

$\therefore x = 3$ or $x = 4$

From (1) : $\therefore y = 4$ or $y = 3$

$\therefore$ The S.S. $= \{(3, 4), (4, 3)\}$

Find in $\mathbb{R}$ the S.S. of each of the following equations using the general formula :

14 $x^2 - 4x + 1 = 0$

Solution

$\because a = 1 , b = -4 , c = 1$

$\therefore x = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2}$

$\therefore x \approx 0.27$ or $x \approx 3.73$

$\therefore$ The S.S. $= \{0.27 , 3.73\}$

15 $x^2 = 6x - 7$

Solution

$\because x^2 - 6x + 7 = 0$

$\therefore a = 1 , b = -6 , c = 7$

$\therefore x = \frac{6 \pm \sqrt{36-28}}{2} = \frac{6 \pm \sqrt{8}}{2}$

$\therefore x \approx 1.586$ or $x \approx 4.414$

$\therefore$ The S.S. $= \{1.586 , 4.414\}$

16 $(x-3)^2 - 5x = 0$

Solution

$\because x^2 - 11x + 9 = 0$

$\therefore a = 1 , b = -11 , c = 9$

$\therefore x = \frac{11 \pm \sqrt{121-36}}{2} = \frac{11 \pm \sqrt{85}}{2}$

$\therefore x \approx 10.110$ or $x \approx 0.890$

$\therefore$ The S.S. $= \{10.110 , 0.890\}$

17 $\frac{x}{3} = \frac{1}{5-x}$

Solution

$\because -x^2 + 5x - 3 = 0$

$\therefore a = -1 , b = 5 , c = -3$

$\therefore x = \frac{-5 \pm \sqrt{25-12}}{-2} = \frac{-5 \pm \sqrt{13}}{-2}$

$\therefore x \approx 4.303$ or $x \approx 0.697$

$\therefore$ The S.S. $= \{4.303 , 0.697\}$

18 $\frac{8}{x^2} + \frac{1}{x} = 1$

Solution

$\because -x^2 + x + 8 = 0$

$\therefore a = -1 , b = 1 , c = 8$

$\therefore x = \frac{-1 \pm \sqrt{1+32}}{-2} = \frac{-1 \pm \sqrt{33}}{-2}$

$\therefore x \approx -2.372$ or $x \approx 3.372$

$\therefore$ The S.S. $= \{-2.372 , 3.372\}$

19

Applications

The sum of two natural numbers is 63 and their difference is 11 **Find the two numbers.**

Solution

Let the two numbers be x and y

$\therefore x + y = 63$ (1) , $x - y = 11$ (2)

Adding (1) and (2) : $\therefore 2x = 74$

$\therefore x = 37$

Substituting in equ. (1) : $\therefore y = 26$

$\therefore$ The two numbers are 37 , 26

20

A rectangle is with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. **Find the area of the rectangle.**

Solution

Let the length x cm. and the width be y cm.

$\therefore x - y = 4$ (1) , $2(x + y) = 28$ $\quad \therefore x + y = 14$ (2)

Adding (1) and (2) : $\therefore 2x = 18$ $\quad \therefore x = 9$

Substituting in (1) : $\therefore y = 5$

$\therefore$ The length = 9 cm. , the width = 5 cm.

$\therefore$ The area of the rectangle $= 9 \times 5 = 45$ cm^2

21

A 2-digit number , its tens digit is twice its units digit.
If the two digits are reversed , the resulting number decreases the original number by 27. **Find the original number.**

Solution

	Units	Tens	The number
The original number	x	y	$x + 10y$
The resulting number	y	x	$y + 10x$

$\because$ The resulting number decreases the original number by 27

$\therefore$ The original number – the resulting number = 27

$\therefore (x + 10y) - (y + 10x) = 27$ $\qquad$ $\therefore x + 10y - y - 10x = 27$

$\therefore 9y - 9x = 27$ $\qquad$ $\therefore y - x = 3$ $\qquad$ (2)

Substituting with the value of $y = 2x$ from equation (1) in equation (2) :

$\therefore 2x - x = 3$ $\qquad$ $\therefore \boxed{x = 3}$

Substituting by $x = 3$ in equation (1) :

$\therefore y = 2 \times 3$ $\qquad$ $\therefore \boxed{y = 6}$

$\therefore$ The units digit = 3 , the tens digit = 6 $\qquad$ $\therefore$ The original number = 63

22

The sum of two real numbers is 9 and the difference between their squares equals 45

Find the two numbers.

Solution

Let the two numbers be x and y :

$\therefore x + y = 9$ $\qquad$ (1)

$, x^2 - y^2 = 45$ $\qquad$ (2)

From (1) : $\therefore x = 9 - y$ $\qquad$ (3)

Substituting from (3) in (2) : $\therefore (9 - y)^2 - y^2 = 45$

$\therefore 81 - 18y + y^2 - y^2 = 45$ $\qquad$ $\therefore 81 - 18y = 45$

$\therefore 18y = 36$ $\qquad$ $\therefore y = 2$

Substituting in (3) : $\therefore x = 9 - 2 = 7$

$\therefore$ The two numbers are 7 and 2

23

The perimeter of a rectangle is 18 and its area is 18 cm^2

Find its two dimensions.

Solution

Let the length of the rectangle = x cm.
and the width = y cm.

$\therefore (x + y) \times 2 = 18$ $\qquad$ $\therefore x + y = 9$ (1)

$, xy = 18$ $\qquad$ (2)

From (1) : $\therefore y = 9 - x$ $\qquad$ (3)

Substituting in (2) : $\qquad$ $\therefore x(9 - x) = 18$

$\therefore 9x - x^2 = 18$ $\qquad$ $\therefore x^2 - 9x + 18 = 0$

$\therefore (x - 3)(x - 6) = 0$ $\qquad$ $\therefore x = 3$ or $x = 6$

Substituting in (3) : $\qquad$ $\therefore y = 6$ or $y = 3$

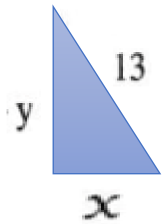
$\therefore$ The two dimensions are 6 cm. and 3 cm.

24

A right-angled triangle of hypotenuse length 13 cm. and its perimeter is 30 cm.

Find the lengths of the other two sides.

Solution



Let the lengths of the two sides of the right angle be x cm. and y cm.

$\therefore x + y + 13 = 30$ $\qquad$ $\therefore x + y = 17$ $\qquad$ (1)

$, \because x^2 + y^2 = 169$ $\qquad$ (2)

From (1) : $\therefore x = 17 - y$ $\qquad$ (3)

Substituting in (2) : $\therefore (17 - y)^2 + y^2 = 169$

$\therefore y^2 - 34y + 289 + y^2 - 169 = 0$

$\therefore 2y^2 - 34y + 120 = 0$ $\qquad$ $\therefore y^2 - 17y + 60 = 0$

$\therefore (y - 12)(y - 5) = 0$ $\qquad$ $\therefore y = 12$ or $y = 5$

Substituting in (3) : $\therefore x = 5$ or $x = 12$

$\therefore$ The side lengths of the right angle are 5 cm. and 12 cm.

25

A length of a rectangle is 3 cm. more than its width and its area is 28 cm^2.
Find its perimeter.

Solution

Let the length of the rectangle be x cm and its width be y cm.

$\therefore x - y = 3$ (1)

$, x y = 28$ (2)

From (1) : $\therefore x = y + 3$ (3)

Substituting from (3) in (2) :

Substituting from (3) in (2) :

$\therefore y(y+3) = 28 \qquad \therefore y^2 + 3y - 28 = 0$

$\therefore (y+7)(y-4) = 0$

$\therefore y = -7$ (refused) or $y = 4$

Substituting in (3) : $\therefore x = 7$

$\therefore$ The two dimensions of the rectangle are 4 cm. and 7 cm.

$\therefore$ The perimeter of the rectangle $= (7+4) \times 2 = 22$ cm.

26

The sum of two rational numbers is 12 , and three times the smallest number exceeds than twice the greatest number by one Find the two numbers.

Solution

Let the two numbers be x and y :

$\therefore x + y = 12 \qquad \therefore y = 12 - x$ (1)

$3x - 2y = 1$ (2)

Substituting from (1) in (2) :

$3x - 2(12 - x) = 1 \qquad \therefore 3x - 24 + 2x = 1$

$\therefore 5x - 24 = 1 \qquad \therefore 5x = 25 \qquad \therefore x = 5$

Substituting in (1) : $\therefore y = 7$

$\therefore$ The two numbers are 5 and 7

27

Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations graphically :

$y = x + 1 \quad , \quad 2x + y = 7$

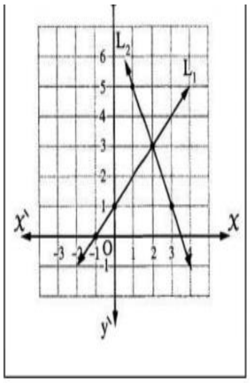
Solution

$y = x + 1$

x	-1	0	2
y	0	1	3

$y = 7 - 2x$

x	1	2	3
y	5	3	1



From the graph : The S.S. $= \{(2, 3)\}$

28

Find in $\mathbb{R}$ the set of zeroes of the functions that are defined by the following rules :

❶ $f(x) = 3x^2 - 15x$

❷ $f(x) = x^2 + 4$

❸ $f(x) = \frac{x^2 - 9}{x^2 - x - 6}$, then find the domain of f

❹ $f(x) = \text{zero}$, $n(x) = 5$

Solution

❶ Let $3x^2 - 15x = 0$

$\therefore 3x(x - 5) = 0$

$\therefore x = 0$ or $x = 5$

$\therefore z(f) = \{0, 5\}$

❷ Let $x^2 + 4 = 0$

$\therefore x^2 = -4 \quad \therefore x = \pm\sqrt{-4}$

$, \because \sqrt{-4} \notin \mathbb{R}$

$\therefore z(f) = \varnothing$

❸ $\because f(x) = \frac{(x-3)(x+3)}{(x-3)(x+2)}$

$\therefore$ The domain of $f = \mathbb{R} - \{3, -2\}$, $z(f) = \{3, -3\} - \{3, -2\} = \{-3\}$

❹ $z(f) = \mathbb{R}$, $z(n) = \varnothing$

29

If the domain of the function n : $n(x) = \frac{x-1}{x^2 - ax + 9}$ is $\mathbb{R} - \{3\}$, then find the value of a

Solution

$\because$ The domain of n $= \mathbb{R} - \{3\}$

$\therefore$ At $x = 3$, then $x^2 - ax + 9 = 0$

$\therefore 9 - 3a + 9 = 0 \quad \therefore 3a = 18 \quad a=6$

30

If the set of zeroes of the function $f(x) = ax^2 + x + b$ is $\{0, 1\}$

Find the value of a and b

Solution

$\because z(f) = \{0, 1\} \quad \therefore f(0) = 0$

$\therefore b = 0 \quad \therefore f(x) = ax^2 + x$

$, \because f(1) = 0 \quad \therefore a \times 1^2 + 1 = 0$

$\therefore a + 1 = 0 \quad \therefore a = -1$

31

If the function $f : f(x) = x^3 - 2x^2 - 75$

Prove that : The number 5 is the one of the zeroes of the function f

Solution

$\because f(5) = (5)^3 - 2(5)^2 - 75 = 125 - 50 - 75 = 0$

$\therefore$ the number 5 is one of zeroes of the function f

32

Find the common domain of the following algebraic fractions :

$$\frac{x+2}{x+5} , \frac{x-4}{x-7}$$

Solution

The domain of $n_1 = \mathbb{R} - \{-5\}$

, the domain of $n_2 = \mathbb{R} - \{7\}$

$\therefore$ The common domain =

$= \mathbb{R} - \{-5, 7\}$

33

Find the common domain of the following algebraic fractions :

$$\frac{x}{x^2 - 4} , \frac{3}{2 - x}$$

Solution

$\because n_1(x) = \frac{x}{(x+2)(x-2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-2, 2\}$

The domain of $n_2 = \mathbb{R} - \{2\}$

$\therefore$ The common domain =

$\mathbb{R} - \{-2, 2\}$

34

Find the common domain of the following algebraic fractions :

$$\frac{x}{3} , \frac{3}{x}$$

Solution

The domain of n_1 $\mathbb{R}$

the domain of $n_2 = \mathbb{R} - \{0\}$

$\therefore$ The common domain $= \mathbb{R} - \{0\}$

35

If the domain of the function f where $f(X) = \frac{X+b}{X+a}$ is $\mathbb{R} - \{-2\}$ and $f(0) = 3$, then find the value of each a and b

Solution

$\because$ The domain $= \mathbb{R} - \{-2\}$

$\therefore$ When $X = -2$ $\quad \therefore X + a = 0$

$\therefore -2 + a = 0$ $\quad \therefore a = 2$

$\therefore f(X) = \frac{X+b}{X+2}$, $\because f(0) = 3$ $\quad \therefore \frac{0+b}{0+2} = 3$

$\therefore \frac{b}{2} = 3$ $\quad \therefore b = 6$

36

If the set of zeroes of the function f where $f(X) = \frac{a X^2 - 6X + 8}{bX - 4}$ is $\{4\}$ and its domain is $\mathbb{R} - \{2\}$, then find a , b

Solution

$\because z(f) = \{4\}$ $\quad \therefore$ At $X = 4$

$\therefore a X^2 - 6X + 8 = 0$

$\therefore a \times 4^2 - 6 \times 4 + 8 = 0$

$\therefore 16a - 16 = 0$ $\quad \therefore 16a = 16$ $\quad \therefore a = 1$

, $\because$ The domain of $f = \mathbb{R} - \{2\}$

$\therefore$ At $X = 2$ $\quad \therefore bX - 4 = 0$

$\therefore 2b - 4 = 0$ $\quad \therefore 2b = 4$ $\quad \therefore b = 2$

37

If the domain of the function f where $f(X) = \frac{X}{X^2 - 5X + m}$ is $\mathbb{R} - \{2, c\}$, then find the value of each m and c

Solution

$\because$ The domain of $f = \mathbb{R} - \{2, c\}$

$\therefore$ When $X = 2$ $\quad \therefore X^2 - 5X + m = 0$

$\therefore 4 - 5 \times 2 + m = 0$ $\quad \therefore m = 6$

$\therefore f(X) = \frac{X}{X^2 - 5X + 6}$ $\quad \therefore f(X) = \frac{X}{(X-2)(X-3)}$

$\therefore$ The domain of $f = \mathbb{R} - \{2, 3\}$ $\quad \therefore c = 3$

38

prove that $n_1 = n_2$:

$n_1(X) = \frac{2X}{2X+4}$, $n_2(X) = \frac{X^2 + 2X}{X^2 + 4X + 4}$

Solution

$\because n_1(X) = \frac{2X}{2(X+2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-2\}$

, $n_1(X) = \frac{X}{X+2}$ $\quad$ (1)

$\because n_2(X) = \frac{X(X+2)}{(X+2)^2}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-2\}$

, $n_2(X) = \frac{X}{X+2}$ $\quad$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

39

If $n_1(X) = \frac{X^2 - 4}{X^2 + X - 6}$, $n_2(X) = \frac{X^3 - X^2 - 6X}{X^3 - 9X}$

Prove that : $n_1(X) = n_2(X)$

for all values of X which belong to the common domain and find this domain.

Solution

$\because n_1(X) = \frac{(X-2)(X+2)}{(X+3)(X-2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-3, 2\}$

, $n_1(X) = \frac{X+2}{X+3}$ $\quad \because n_2(X) = \frac{X(X+2)(X-3)}{X(X+3)(X-3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, -3, 3\}$

, $n_2(X) = \frac{X+2}{X+3}$ $\quad \therefore n_1(X) = n_2(X)$

For all the values of $X \in \mathbb{R} - \{0, -3, 2, 3\}$

40

, prove that $n_1 = n_2$:

$n_1(x) = \frac{x^2}{x^3 - x^2}$ $\quad n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$

Solution

$\because n_1(x) = \frac{x^2}{x^2(x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0 , 1\}$

$, n_1(x) = \frac{1}{x-1}$ $\quad$ (1)

$\because n_2(x) = \frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0 , 1\}$

$, n_2(x) = \frac{1}{x-1}$ $\quad$ (2)

From (1) and (2) : $\therefore n_1 = n_2$

41

If the domain of the function n where

$n(x) = \frac{b}{x} + \frac{9}{x+a}$ is $\mathbb{R} - \{0 , 3\}$

n (6) = 7 find the values of a , b

Solution

$\because$ The domain of $n = \mathbb{R} - \{0 , 3\}$

$\therefore$ At $x = 3$ $\quad \therefore x + a = 0$

$\therefore 3 + a = 0$ $\quad \therefore a = -3$

$\therefore n(x) = \frac{b}{x} + \frac{9}{x-3}$

$, \because n(6) = 7$ $\quad \therefore \frac{b}{6} + \frac{9}{6-3} = 7$

$\therefore \frac{b}{6} + 3 = 7$ $\quad \therefore \frac{b}{6} = 4$ $\quad \therefore b = 24$

42

Find n (x) in its simplest form

$n(x) = \frac{x^2 + x}{x^2 - 1} - \frac{x+5}{x^2 + 4x - 5}$

Solution

$n(x) = \frac{x(x+1)}{(x+1)(x-1)} - \frac{x+5}{(x-1)(x+5)}$

The domain of $n = \mathbb{R} - \{-1 , 1 , -5\}$

$n(x) = \frac{x-1}{x-1} = 1$

43

$n(x) = \frac{x^2 + 2x + 4}{x^3 - 8} - \frac{9 - x^2}{x^2 + x - 6}$

Solution

$n(x) = \frac{x^2 + 2x + 4}{(x-2)(x^2+2x+4)} - \frac{(3-x)(3+x)}{(x-2)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -3\}$

$, n(x) = \frac{1}{x-2} - \frac{3-x}{x-2} = \frac{x-2}{x-2} = 1$

44

$n(x) = \frac{x^3 - 8}{x^2 - 3x + 2} \times \frac{x+1}{x^2 + 2x + 4}$

Solution

$n(x) = \frac{(x-2)(x^2+2x+4)}{(x-2)(x-1)} \times \frac{x+1}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , 1\}$

$, n(x) = \frac{x+1}{x-1}$

Solution

45

$$n(x) = \frac{3}{x+1} + \frac{2x+1}{1-x^2}$$

Solution

$\because n(x) = \frac{3}{x+1} + \frac{2x+1}{(1-x)(1+x)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-1, 1\}$

, $n(x) = \frac{3(1-x)+2x+1}{(1+x)(1-x)} = \frac{4-x}{(1+x)(1-x)}$

46

$$\frac{x^3 - x^2 - 2x}{x^2 - 5x + 6} \times \frac{x^2 + 2x - 15}{x^3 + 6x^2 + 5x} :$$

then find n (7) , n (3) if possible.

Solution

$\because n(x) = \frac{x(x-2)(x+1)}{(x-2)(x-3)} \times \frac{(x-3)(x+5)}{x(x+5)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, 3, -5, 0, -1\}$

, $n(x) = 1$ $\therefore n(7) = 1$

$\therefore n(3)$ undefined because $3 \notin$ the domain of n

47

If $f_1(x) = \frac{x-a}{x+b}$, and the set of zeroes of f_1 is $\{5\}$, and the domain of f_1 is $\mathbb{R} - \{3\}$, then find the values of a and b

If $f_2(x) = \frac{x-1}{x-3}$, then find $f_1(x) + f_2(x)$ in the simplest form.

Solution

$\because$ The set of zeroes of $f_1(x)$ is $\{5\}$

$\therefore 5 - a = 0$ $\therefore a = 5$

, $\because$ the domain of $f_1(x) = \mathbb{R} - \{3\}$

$\therefore 3 + b = 0$ $\therefore b = -3$

$\therefore f_1(x) + f_2(x) = \frac{x-5}{x-3} + \frac{x-1}{x-3}$

$= \frac{2x-6}{x-3} = \frac{2(x-3)}{x-3} = 2$

47

$$n(x) = \frac{x^2 - 3x + 2}{x^2 - 1} \div \frac{3x - 15}{x^2 - 4x - 5}$$

Solution

$\because n(x) = \frac{(x-2)(x-1)}{(x-1)(x+1)} \div \frac{3(x-5)}{(x+1)(x-5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1, -1, 5\}$

, $n(x) = \frac{x-2}{x+1} \times \frac{x+1}{3} = \frac{x-2}{3}$

48

$$n(x) = \frac{x^2 + 2x + 1}{2x - 8} \times \frac{x-4}{x+1}$$

Solution

$\because n(x) = \frac{(x+1)^2}{2(x-4)} \times \frac{x-4}{x+1}$

$\therefore$ The domain of $n = \mathbb{R} - \{-1, 4\}$

, $n(x) = \frac{x+1}{2}$

49

If the domain of n : $n(x) = \frac{\ell}{x} + \frac{9}{x+m}$

$\mathbb{R} - \{0, -2\}$, $n(4) = 1$ **Find :** ℓ , m

Solution

$\because$ The domain of $n = \mathbb{R} - \{0, -2\}$

$\therefore$ At $x = -2$ $\therefore x + m = 0$

$\therefore -2 + m = 0$ $\therefore m = 2$

$\therefore n(x) = \frac{\ell}{x} + \frac{9}{x+2}$

, $\because n(4) = 1$ $\therefore \frac{\ell}{4} + \frac{9}{4+2} = 1$

$\therefore \frac{\ell}{4} + \frac{9}{6} = 1$ $\therefore \frac{\ell}{4} = -\frac{1}{2}$ $\therefore \ell = -\frac{4}{2} = -2$

50

If $n(x) = \dfrac{x-1}{x+3}$ find $n^{-1}(x)$ and $n^{-1}(x)$ and identify the domain of n^{-1}

Solution

$\because n(x) = \dfrac{x-1}{x+3}$ $\quad \therefore n^{-1}(x) = \dfrac{x+3}{x-1}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{1, -3\}$

51

If $n(x) = \dfrac{x^2-3x}{(x-3)(x^2+2)}$

then find : $n^{-1}(x)$ in the simplest form showing the domain of n^{-1}

Solution

$\because n(x) = \dfrac{x(x-3)}{(x-3)(x^2+2)}$

$\therefore n^{-1}(x) = \dfrac{(x-3)(x^2+2)}{x(x-3)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 3\}$

$, n^{-1}(x) = \dfrac{x^2+2}{x}$

52

Find the set of zeroes of the function f

$f(x) = \dfrac{x-1}{x+1}$, then find $f^{-1}(2)$

Solution

$z(f) = \{1\}$ $\quad , \because f^{-1}(x) = \dfrac{x+1}{x-1}$

$\therefore$ The domain of $f^{-1} = \mathbb{R} - \{-1, 1\}$

$, f^{-1}(2) = \dfrac{2+1}{2-1} = \dfrac{3}{1} = 3$

53

If $n(x) = \dfrac{x^2+3x}{x^2+x-6}$

(1) **Find :** $n^{-1}(x)$ and find the domain of n^{-1}

(2) If $n^{-1}(x) = 2$, find value of x

Solution

(1) $\because n(x) = \dfrac{x(x+3)}{(x+3)(x-2)}$

$\therefore n^{-1}(x) = \dfrac{(x+3)(x-2)}{x(x+3)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, -3, 2\}$

$, n^{-1}(x) = \dfrac{x-2}{x}$

(2) $\because n^{-1}(x) = 2$ $\quad \therefore \dfrac{x-2}{x} = 2$

$\therefore x-2 = 2x$ $\quad \therefore x = -2$

54

If the fraction $\dfrac{x+2}{x^2-4}$ is the multiplicative inverse of of $\dfrac{x-2}{h}$ where $x \notin \{2, -2\}$, then calculate h

Solution

$\because n(x) = \dfrac{x+2}{(x-2)(x+2)}$

$\therefore n^{-1}(x) = \dfrac{(x-2)(x+2)}{x+2} = x-2$

$\because n^{-1}(x) = \dfrac{x-2}{h}$ $\quad \therefore h = 1$

55

$n(x) = \dfrac{x-1}{x+3}$ find $n^{-1}(x)$ and identify the domain of n^{-1}

Solution

$\because n(x) = \dfrac{x-1}{x+3}$ $\quad \therefore n^{-1}(x) = \dfrac{x+3}{x-1}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{1, -3\}$

56

If A and B are two events of the sample space of a random experiment , $P(A) = \frac{1}{5}$, $P(B) = \frac{3}{5}$ and $P(A \cap B) = \frac{1}{10}$ **Find :**

(1) $P(A')$ (2) $P(B')$ (3) $P(A \cup B)$

(4) $P(A - B)$ (5) $P(B - A)$

Solution

(1) $P(A') = 1 - P(A) = 1 - \frac{1}{5} = \frac{4}{5}$

(2) $P(B') = 1 - P(B) = 1 - \frac{3}{5} = \frac{2}{5}$

(3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{5} + \frac{3}{5} - \frac{1}{10} = \frac{7}{10}$

(4) $P(A - B) = P(A) - P(A \cap B) = \frac{1}{5} - \frac{1}{10} = \frac{1}{10}$

(5) $P(B - A) = P(B) - P(A \cap B) = \frac{3}{5} - \frac{1}{10} = \frac{1}{2}$

57

If X and Y are two events of a sample space S , $P(X) = 0.35$, $P(Y) = 0.48$ and $P(X \cup Y) = 0.6$ **Find :**

(1) $P(X')$, $P(Y')$ (2) $P(X \cap Y)$

(3) $P(X - Y)$ (4) $P(X \cap Y)'$

Solution

(1) $P(X') = 1 - P(X) = 1 - 0.35 = 0.65$

$P(Y') = 1 - P(Y) = 1 - 0.48 = 0.52$

(2) $\because P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$\therefore P(X \cap Y) = P(X) + P(Y) - P(X \cup Y)$

$= 0.35 + 0.48 - 0.6 = 0.23$

(3) $P(X - Y) = P(X) - P(X \cap Y) = 0.35 - 0.23 = 0.12$

(4) $P(X \cap Y)' = 1 - P(X \cap Y) = 1 - 0.23 = 0.77$

58

If A and B are two events of a sample space of a random experiment , $P(B) = \frac{1}{3}$ and $P(A - B) = \frac{1}{4}$ **Find : P (A) if :**

(1) $P(A \cap B) = \frac{1}{12}$ (2) A and B are mutually exclusive. (3) $B \subset A$

Solution

(1) $P(A) = P(A - B) + P(A \cap B) = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$

(2) $P(A) = P(A - B) = \frac{1}{4}$

(3) $\because B \subset A$ $\therefore P(A \cap B) = P(B) = \frac{1}{3}$

$\therefore P(A) = P(A - B) + P(A \cap B) = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$

59

If A and B are two events from the sample space of a random experiment , $P(A) = 0.8$, $P(B) = 0.7$ and $P(A \cap B) = 0.6$ **Find :**

(1) The probability of non occurrence of the events A and B together.

(2) The probability of occurrence of at least one of the two events.

Solution

(1) The probability of non occurrence the two events A and B together $= P(A \cap B)' = 1 - P(A \cap B)$

$= 1 - 0.6 = 0.4$

(2) The probability of occurrence of one of the two events at least $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.8 + 0.7 - 0.6 = 0.9$

60

If A and B are two events of a sample space of a random experiment , $P(A) = 0.5$, $P(B) = 0.6$ and $P(A \cap B) = 0.4$ **Find :**

(1) The probability of occurrence at least one of the two events.

(2) The probability of occurrence of B and non occurrence of A

(3) The probability of non occurrence of A

(4) The probability of non occurrence of any of them.

(5) The probability of occurrence of one of the events but not the other.

(6) The probability of occurrence of the event A only.

Solution

(1) The probability of occurrence of one of the two events at least $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.6 - 0.4 = 0.7$

(2) The probability of occurrence of the event B and non occurrence the event A

$= P(B - A) = P(B) - P(A \cap B) = 0.6 - 0.4 = 0.2$

(3) The probability of non occurrence of the event A

$= P(A') = 1 - P(A) = 1 - 0.5 = 0.5$

(4) The probability of non occurrence of any one of the two events $= P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.7 = 0.3$

(5) The probability of occurrence of one of the two events but not the other $= P(A - B) + P(B - A)$

$= P(A) + P(B) - 2\,P(A \cap B)$

$= 0.5 + 0.6 - 2 \times 0.4 = 0.3$

(6) The probability of occurrence of the event A only

$= P(A - B) = P(A) - P(A \cap B) = 0.5 - 0.4 = 0.1$

61

If A and B are two events from a sample space of a random experiment , and $P(A) = 0.7$, $P(B) = 0.6$, $P(A \cap B) = 0.4$, **then find :**

(1) $P(A \cup B)$ (2) $P(A - B)$

Solution

(1) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.6 - 0.4 = 0.9$

(2) $P(A - B) = P(A) - P(A \cap B) = 0.7 - 0.4 = 0.3$

62

If A and B are two events in a sample space for a random experiment , and if $P(A) = 0.8$, $P(B) = 0.7$ and $P(A \cap B) = 0.6$

Find : (1) The probability of non occurrence of the event A

(2) The probability of occurrence one of the two events at least.

Solution

(1) The probability of non occurence of the event A $= p(\acute{A}) = 1 - P(A) = 1 - 0.8 = 0.2$

(2) The probability of occurence of the two events at least $= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B) = 0.8 + 0.7 - 0.6 = 0.9$

63

If A and B are two events of the sample space of a random experiment $P(A) = \frac{5}{9}$, $P(B) = \frac{2}{9}$, $P(A \cap B) = \frac{1}{9}$

Find : (1) $P(A \cup B)$

(2) The probability of non occurrence any of the two events.

(3) The probability of occurrence of event A only.

Solution

(1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{9} + \frac{2}{9} - \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$

(2) The probability of non occurrence any of the two events $= P(\acute{(A \cup B)})$

$= 1 - P(A \cup B) = 1 - \frac{2}{3} = \frac{1}{3}$

(3) The probability of occurrence of event A only

$= P(A - B) = P(A) - P(A \cap B) = \frac{5}{9} - \frac{1}{9} = \frac{4}{9}$

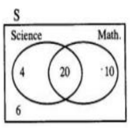
A classroom consists of 40 students , 30 of them succeeded in math. 24 in science and 20 in both math. and science. If a student is chosen randomly.

Find the probability that this student is :

(1) fail in math. (2) succeeded in math. or science

Solution

(1) The probability that the chosen student fail in Math. $= \frac{10}{40} = \frac{1}{4}$



(2) The probability that the chosen student succeeded in Math. or Science $= \frac{34}{40} = \frac{17}{20}$

64

If A , B are two events in a random experiment where :

$P(A) = 0.7$, $P(B) = 0.6$, $P(A \cap B) = 0.3$

Calculate the value of : (1) $P(\acute{A})$ (2) $P(A - B)$ (3) $P(A \cup B)$

Solution

(1) $P(\acute{A}) = 1 - P(A) = 1 - 0.7 = 0.3$

(2) $P(A - B) = P(A) - P(A \cap B) = 0.7 - 0.3 = 0.4$

(3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.6 - 0.3 = 1$

64

A box contains 30 identical cards numbered from 1 to 30 and a card was drawn randomly.

Calculate the probability that the number on the drawn card is :

(1) Divisible by 4 (2) A prime number.

Solution

The probability that the number on the drawn card is divisible by 4 $= \frac{7}{30}$

The probability that the number on the drawn card is a prime number $= \frac{10}{30} = \frac{1}{3}$

64

If A and B are two events from the sample space of a random experiment where $P(B) = \frac{1}{12}$, $P(A \cup B) = \frac{1}{3}$

Find P (A) in each of the following cases :

(1) A and B mutually exclusive. (2) $B \subset A$

Solution

$\because$ A and B are mutually exclusive events

$\therefore P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B)$ $\therefore P(A) = P(A \cup B) - P(B) = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$

(2) $\because B \subset A$ $\therefore P(A \cup B) = P(A) = \frac{1}{3}$

64

A bag contains 21 symmetrical balls , 8 white , 6 red and the rest is black , one ball was drawn randomly , find the probability that it was :

(1) White. (2) Not black. (3) Red or black.

Solution

(1) The probability that it was white $= \frac{8}{21}$

(2) The probability that it was not black

$= \frac{8+6}{21} = \frac{14}{21} = \frac{2}{3}$

(3) The number of black balls $= 21 - (8 + 6) = 7$

The probability that it was red or black

$= \frac{6+7}{21} = \frac{13}{21}$