Pure math – Model 1

1. If $(1, \omega, \omega^2)$ are the cubic roots of 1, then $(\omega + \omega^2 + \cdots + \omega^{100}) = \cdots$
a) 1   b) ω c) ω^2 d) zero

2. If θ, θ, β are directed angles of $\vec{A}$ and $\sin^2 \beta = 3\sin^2 \theta$, then $\cos^2 \theta = \cdots$
a) $\frac{3}{5}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{2}$

3. If $n = \ln x, y = e^n$, then $\frac{dy}{dx} = \cdots$
a) Zero   b) 1   c) 2   d) 3

4. $\int 6xe^{3x^2+1} \cdot dx = \cdots + c$
a) e^{x^2+1} b) e^{3x^2} c) e^{3x^2+1} d) $\frac{1}{e^{x^2+1}}$

5. In the expansion of $(3 + 2x)^8 + (3 - 2x)^8$ at $x = \frac{1}{6}$, Then middle term $= \cdots$
a) 110   b) 120   c) 130   d) 140

6. If the point $(k, 4,5)$ is at equal distances from the x and z axes, then $k = \cdots$
a) ± 1 b) ± 3 c) ± 4 d) ± 5

7. If $x^2 y^3 = 8$, then $\frac{dy}{dx} = \cdots$ at $x = -1$
a) $\frac{4}{3}$ b) $\frac{-4}{3}$ C) $\frac{3}{4}$ d) $\frac{1}{2}$

8. $\int \frac{(\ln x)^2}{x} dx = \cdots + c$
a) $\frac{1}{3}(\ln x)^3$ b) $\frac{1}{2}(\ln x)^3$ c) $\ln x$ d) $\ln x^2$

9. If the middle term in the expansion of $(1 + x)^{10}$ is twice the seventh term, then $x = \cdots$

a) 0.2   b) 0.4   c) 0.6   d) 0.8

10. If $\overrightarrow{AB} = -3\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$, and $\overrightarrow{BC} = \hat{\jmath} + 5\hat{k}$, then $\| \overrightarrow{AC} \| = \cdots$

a) 8   b) 10   c) 12   d) 13

11. If $x = 3t^2 - 1, y = t^3 + 2$, then $\frac{d^2y}{dx^2} = \cdots$ at $t = 4$

a) 48   b) $\frac{1}{24}$ c) $\frac{1}{48}$ d) 24

12. The volume of the solid generated by rotating the region bounded by the curve $y = x(x - 2)$ a complete cycle about the x-axis $= \cdots$ cubic unit

a) $\frac{16}{15}\pi$ b) $\frac{19}{15}\pi$ c) $\frac{17}{15}\pi$ d) $\frac{15}{17}\pi$

13. The trigonometric form of the complex number $z = \frac{5 - \sqrt{3}i}{\sqrt{3} - 2i}$ is ....

a) $\cos \frac{\pi}{3} + i\sin \frac{\pi}{3}$ b) $\cos \frac{\pi}{4} + i\sin \frac{\pi}{4}$

c) $2\left(\cos \frac{\pi}{6} + i\sin \frac{\pi}{6}\right)$ d) $3\left(\cos \frac{\pi}{3} + i\sin \frac{\pi}{3}\right)$

14. The equation of the plane passing through the point $(1,2,3)$ and parallel to both the x and y axes is ...

a) $x + y = 3$ b) $x = 1$ c) $y = 2$ d) $z = 3$

15. A point is moving according to the relation $S = 3t^3 + 3t^2 - 4$, then $\frac{ds}{dt} = \cdots$ at $t = 3$

a) 77   b) 88   c) 99   d) 111

16. The two square roots of the number $z = 3 + 4i$ is ...
a) $\pm(2 + i)$ b) $\pm(2 + \sqrt{3}i)$
c) $\pm(1 + \sqrt{3}i)$ d) $\pm(1 + i)$

17. The direction vector of the straight line $\frac{x-2}{3} = \frac{y+3}{2}, z = 4$ is ...
a) $(3,2,4)$ b) $(3,2,0)$ c) $(2,-3,4)$ d) $(2,-3,0)$

18 If $f: f(x) = \sqrt[3]{x^2 - 6x}$, then the number of critical points of the curve of f is ...
a) Zero   b) 1   c) 2   d) 3

Essay Questions:

19. If $k \in R$, then find the value of
$$\left(k - \frac{k+1}{\omega+1} + \omega^2(k + 1)\right)^8$$

20. The perimeter of a circular sector is 30 cm, find its radius when its area is maximum.